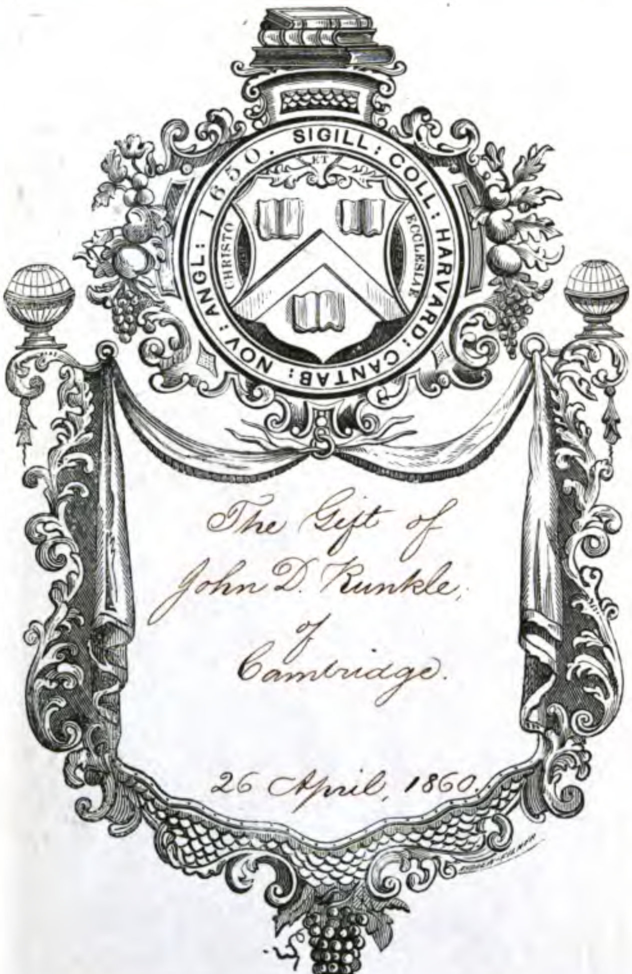
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CHRISTO ET ECCLESIAE
The Gift of
John D. Runkle,
of
Cambridge.
26 April, 1860.

Yaxon, Alfred 13.
254

REPORT

ON

WEIGHTS AND MEASURES

READ BEFORE THE

PHARMACEUTICAL ASSOCIATION

AT THEIR

EIGHTH ANNUAL SESSION

HELD IN BOSTON SEPTEMBER 15, 1859.

BY

ALFRED B. TAYLOR,

OF PHILADELPHIA,

CHAIRMAN OF THE COMMITTEE ON WEIGHTS AND MEASURES.

BOSTON:
PRESS OF GEO. C. RAND & AVERY,
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REPORT.

THE Committee on Weights and Measures has given the subject referred to it a very careful and deliberate consideration. So grave, however, are the inconveniences attending any important change in the established system, and so varied and complex are the conditions to be fulfilled by any standards and divisions which shall deserve to displace it, that your Committee has with every step of its progress in the investigation felt only an increased sense of the difficulty and responsibility of the task. The general attention given to the question of a reform in weights and measures in all departments of trade, and among almost all classes of men, not only in our own country but in England, sufficiently attests the need that is felt for such reform; while the great diversity of opinion expressed in the discussions flowing out of this agitation, and the number and incongruity of the projects suggested as remedies for the existing evils, equally well attest the intricacy of the mechanism, and the insufficiency of all the schemes thus far proposed for its reorganization.

It appears strange that while our calculations are universally made by a decimal system of arithmetic, (that is, by groupings of ten in a series of ascending orders,) not one of our various tables of weights or measures is based upon this scale, nor has even one of the multitudinous subdivisions of either of them by accident fallen upon the number 10; although almost every other number has been called into requisition, including various fractions.*

* Thus 1½ tierces make 1 hogshead (of wine measure); 2¼ inches make one nail (of cloth measure); 5½ yards, or 16½ feet, make one rod (of land measure); 437½ Troy grains make one ounce avoirdupois; — or 1.0948 oz. Troy are equal to 1 oz. avoirdupois. So incorporating the apothecaries' denominations with the Troy, from which they were derived, we have 1⅕ scruples make one pennyweight, 2½ pennyweights make one drachm, &c.

By this incongruous and irrational system, we are constantly compelled to resort to compound arithmetic, as it is called, to make the most trivial computations, and not unfrequently must reduce a long series of values to their lowest terms, or rather to a common unit, by a process of multiplication and addition, merely to render arithmetical calculations practicable, and then immediately reconvert the resulting amount to the original form, or to its largest units, by the reverse process of division and subtraction.

Still more strange and embarrassing is it (as though these inconveniences were not sufficient for the popular taste,) that so simple an idea as that of weight must be estimated by distinct standards and divisions, — varying with the substance to be weighed; and that the same terms should thus have come to denote entirely different values. As Mr. Adams has well stated in his valuable Report on Weights and Measures, — "The pound avoirdupois is heavier than the pound Troy; but the ounce avoirdupois is lighter than the ounce Troy. The weights and measures of all the old systems present the perpetual paradox of a whole not equal to all its parts." That these great and glaring inconveniences should have been so long tolerated, would appear surprising, did we not consider the wide extent to which the received denominations are employed, and the extreme difficulty of inducing any large numbers of men to abandon that to which they have been long accustomed, for an untried system, which, whatever its promise of advantage, would necessarily involve considerable additional labor, and much confusion in the attempt to adopt it. Indeed, so all-important is uniformity to the successful working of any system of weights and measures, that it may well be doubted whether any abstract excellence of method would justify the introduction of new scales or standards, unless they offered the prospect of a general adoption. This consideration is sufficient to remind the Committee and this Association that we can take no final action in the matter; that the only proper function of the body in this connection is merely to suggest such improvements as may be deemed expedient; — since only a political sanction can give the authoritative establishment to a code of measures, so vital to its efficient working, and its national acceptance.

Different projects in remedy of the existing and acknowledged evils suggest themselves; some more practicable, others more systematic; and unfortunately these two classes appear to bear an inverse ratio to each other.

The most obvious improvement would be, in the first place, a restriction of all weights and measures to a single scale each, for all purposes; — and in the second place, the substitution of decimal divisions, conformably to our established arithmetical notation. The old names might be retained, but with the new values necessarily derived from the changed proportions of the subdivisions; and this would probably be the simplest and most feasible reform which could be suggested, since all experience shows that, however much more rational it may be to assign new terms to new things or ideas, mankind is ever far more ready to adopt a change of substance than of form, and far more prone to transfer old names to new but analogous uses, than to accept an unfamiliar nomenclature expressly contrived for such new uses.* The following table adapting the received subdivisions and terms of apothecaries' and avoirdupois weights to a decimal scale would be sufficiently similar to the existing system, to probably present no great difficulty to general acceptance:

Decimal Table of Weights.

		1 grain	1 =	(Troy Weight) 1 gr.
10 grains	make	1 scruple	10 =	½ scr.
10 scruples	"	1 drachm	100 =	1 dr. 2 scr.
10 drachms	"	1 ounce	1000 =	2 oz. and 2 scr.
10 ounces	"	1 pound	10000 =	1 lb. 8 oz. 6 dr. 2 scr.
10 pounds	"	1 stone	100000 =	17 lbs. 4 oz. 2 dr. 2 scr.
10 stones	"	1 hund. wt.	1000000 =	173 lbs. 7 oz. 2 dr. 2 scr.
10 hund. wt.	"	1 ton	10000000 =	1736 lbs. 1 oz. 2 dr. 2 scr.

* Mr. Adams, speaking of the nomenclature of the French system of measures, remarks: "This is the part of the system which has encountered the most insuperable obstacles in France. The French nation have refused to learn, or to repeat these twelve words. They have been willing to take a total and radical change of things; but they insist upon calling them by old names. They take the metre; but they must call one third part of it a foot. They accept the kilogramme; but instead of pronouncing its name, they choose to call one half of it a pound." — (*Report on Weights and Measures, made to Congress, Feb.* 22, 1821.)

This table, in its divisions at least, has been proposed by various writers who have suggested it, apparently, independently of each other. Considerable differences have however existed among these reformers, as to the standard of weight that should be adopted as the starting point in such a decimal scale. On this important question your Committee feels that too many varied interests are involved, to permit a hasty opinion or any decisive judgment at this time. Unless such a standard can be found as shall ultimately be universally received for all kinds of weight, a change would be productive of more evil than benefit. If we accept the Troy grain as the unit, (and from the great value of this weight in Pharmacy especially, and the vast amount of useful knowledge embodied in it, this would seem to be the one most important to be preserved,) we should have the relative values for each denomination exhibited in the preceding table. By this apportionment the largest weight of the scale, or the new ton, would be about five-eighths of the present ton; or 1428½ lbs. avoirdupois.

A peculiar beauty in the decimal system, in addition to the extreme facility it affords to all arithmetical operations, is the familiar property that it dispenses entirely with "reduction"; all the units contained in any sum whether large or small, being apparent on mere inspection. Thus supposing the value above assigned to the new pound (1 lb. 8 oz. 6 drachms, 2 scruples) were a decimal expression, we might read it at once as 1,862 scruples, or as 186 drachms and 2-10, or as 18 ounces and 62-100, — or as 1 pound and 862-1000, instead of having to make, as now, a special calculation to ascertain each of these results.

The unit of weight above selected as the standard of the table, namely, the Troy grain, appears to us as being, on the whole, preferable to any other of those now in use; but there are no doubt many who would regard the present pound as the most important for preservation and comparison. Accepting this, we should have the following table of relative values: —

Decimal Standard the Avoirdupois Pound.

1 grain	0. 7 gr. =	(Avoirdupois Weight.)	0.0256 dr.
1 scruple	7 grs. =		0.256 "
1 drachm	70 " =		2.56 "
1 ounce	700 " =	1 oz.	9.6 "
1 pound	7000 " =	1 lb.	
1 stone	70000 " =	10 lbs.	
1 hund. wt.	700000 " =	3 qrs. 16 lbs.	
1 ton	7000000 " =	8 cwt. 3 qrs. 20 lbs.	

We have unfortunately no standards of weight in nature, and are therefore compelled always to refer ultimately to some unit of length, in determining or verifying the value of our measure of the former. Thus the standard unit of weight in this country, copied from that of Great Britain, is the Troy pound,* which is defined as the weight of 22.794422 cubic inches of distilled water at its maximum density (temperature 39.83 degrees Fahrenheit,†) barometer standing at 30 inches; the inch of this measurement being derived from the length of a pendulum vibrating seconds in a vacuum, at the level of the sea,—in Greenwich or London. Such a pendulum is divided into 39.1393 inches.

Conformably to this system of decimal subdivision, the table of measures of capacity (dry and liquid) might be regulated as follows, the standard unit being the ordinary pint of the United States.

* The ancient standard of weight in England was the grain,—so called from being measured by the weight of a thoroughly dried grain of wheat, taken from the middle of the ear. The old silver penny was made to weigh 32 such grains,—hence called the penny-weight, the unit of the Tower pound. This new unit of weight was subsequently divided into 24 equal parts, still called grains; which increased value has ever since been retained and known as "Troy weight."

† The temperature at which water is heaviest (or most contracted) is now pretty well ascertained to be much nearer 39° F. than 40° F., the number commonly assigned. In a paper by James P. Joule, and Dr. Lyon Playfair, published in the "*Lond. Ed. and Dubl. Phil. Mag.*," for Jan., 1847, (vol. 30, p. 41.) 39°. 1 Fahr. is given as a mean result of a series of very careful experiments, for the maximum density of water, and as not probably in error the hundredth part of a degree.

Decimal Table of Measures. (*Capacity.*)

		1 minim	=	0.768 ♍.
10 minims	make	1 scruple	=	7.68 ♍.
10 scruples	"	1 drachm	=	1 dr. 16.8 ♍.
10 drachms	"	1 gill	=	1 oz. 4 dr. 48 ♍.
10 gills	"	1 pint	=	(Wine Measure) 1 pint.
10 pints	"	1 gallon,	=	1 gall. 1 quart.
10 gallons	"	1 firkin,	=	12 galls. 2 quarts.
10 firkins	"	1 butt,	=	125 galls. (or 1 butt nearly.)

This scale would be quite sufficient for measuring either liquid or solid bulks, and would advantageously replace the separate and double scales in common use; for since the only idea involved in either liquid or dry measure, is simply that of volume, there is no good reason for modifying the designation of the measure by the varying nature of the substance measured. If a "pint" may express indifferently a measure of table salt or of milk — of sand or of alcohol — equally well may either be measured by a "gallon" or by a "peck." By the United States standard, the "pint" is 28.875 cubic inches, or the gallon 231 cubic inches, which is equal to 58,372.1754 grains of distilled water, at its maximum density, — the barometer 30.

While the Committee is clearly of opinion that measures of capacity are entirely too important and useful to be abolished — as has been recommended by some, the persuasion is equally strong that determination by weight would be advantageously applied to the measure of many articles at present usually determined by volume.

As both weights and measures are necessarily referred to units of length for their standards, it is evidently a very important matter, in all projects for their revision, that the linear scale should be well determined, and its values accurately established. A reform in measures of length is no less needed than in those of weight and capacity; and all are so connected, that the work of amendment, to be complete and efficient, should embrace all, commencing with the table of "long measure," as it is called, as the great starting point.

Even so early in our national existence as the year 1790, the illustrious Jefferson, then Secretary of State, in obedience to a resolution of Congress calling upon the Secretary to propose a plan or plans for establishing uniformity in the currency, weights, and measures of the United States, presented a report recommending a decimal system of metrology, and its derivation from a natural and permanent standard of length.

We think Mr. Jefferson's Report possesses sufficient interest to justify a copious extract; and regarding his decimal project as one of considerable merit, we shall present his tables entire.

"To obtain uniformity in measures, weights, and coins, it is necessary to find some measure of invariable length, with which as a standard, they may be compared. There exists not in nature, as far as has been hitherto observed, a single subject or species of subject accessible to man, which presents one constant and uniform dimension. The globe of the earth itself indeed might be considered as invariable in all its dimensious, and that its circumference would furnish an invariable measure; but no one of its circles, great or small, is accessible to admeasurement through all its parts, and the various trials to measure definite portions of them, have been of such various result as to show there is no dependence on that operation for certainty. Matter, then, by its mere extension furnishing nothing invariable, its motion is the only remaining resource."

Mr. Jefferson then refers to a mean solar rotation of the earth, as furnishing us through the second's pendulum, with a measure of length convenient and reliable, and accessible to every one. He acknowledges that "even the pendulum is not without its uncertainties," and he alludes to the difficulty of its precise determination, as well as the variations in its measure arising from differences of latitude, altitude, barometric and thermometric conditions, and lastly the disturbance of the driving mechanism (in the case of an attached pendulum) — impossible to be exactly determined. He recommends the latitude of forty-five degrees, then recently proposed in Europe, as a suitable point, and the mean temperature of the year at that location.

Instead, however, of taking the ordinary pendulum of 39 inches, he proposes the second's rod of 5 feet, then generally known as Leslie's pendulum rod. A simple straight rod without the bob or ball, suspended at one end, has, as is well known, its centre of oscillation at a distance of two-thirds of its length from its point of suspension; or, in other words, is one half longer than the common loaded pendulum vibrating in the same time. Such a rod vibrating seconds, is 58.72 inches.

"Let the standard of measure, then, be a uniform, cylindrical rod of iron, of such length as in latitude 45°, at the level of the ocean, and in a cellar or other place, the temperature of which does not vary through the year, shall perform its vibrations in small and equal arcs in one second of time. A standard of invariable length being thus obtained, we may proceed to identify by that, the measures, weights, and coins of the United States. But here a doubt presents itself as to the extent of the reformation meditated by the House of Representatives. The experiment made by Congress in the year 1786, by declaring that there should be one money of account and payment through the United States, and that its parts and multiples should be in a decimal ratio, has obtained such general approbation both at home and abroad, that nothing seems wanting but the actual coinage to banish the discordant pounds, shillings, pence and farthings of the different States, and to establish in their stead the new denominations. Is it in contemplation with the House of Representatives to extend a like improvement to our measures and weights, and to arrange them also in a decimal ratio? The facility which this would introduce into the vulgar arithmetic, would unquestionably, be soon and sensibly felt by the whole mass of the people, who would thereby be enabled to compute for themselves whatever they should have occasion to buy, to sell, or to measure; which the present complicated and difficult ratios place beyond their computation for the most part. Or, is it the opinion of the Representatives that the difficulty of changing the established habits of a whole nation opposes an insuperable bar to this improvement? Under this uncertainty the Secretary of State thinks it his duty to submit alternative

plans, that the House may at their will adopt either the one or the other, exclusively, or the one for the present, and the other for a future time, when the public mind may be supposed to have become familiarized to it."

The first plan proposes to retain the existing measures, referring them merely to the uniform natural standard. "Let the standard rod (of 45°) be divided into 587¼ equal parts, and each declared to be a line;—10 lines an inch, 12 inches a foot," &c. In the table of measures of capacity, no change is suggested, except "to fix the gallon to one determinate capacity, as the unit of measure—both wet and dry: for convenience is in favor of abolishing the distinction between wet and dry measures." As a convenient compromise for the gallon, 270 cubic inches is proposed as its measure. In the table of weights, he remarks, "it would be convenient to suppress the pound and ounce Troy, and the drachm and quarter avoirdupois; and to form into one series the avoirdupois pound and ounce, and the Troy pennyweight and grain." Dividing the avoirdupois ounce into 18 pennyweights, the new pennyweight would contain between a third and a quarter of a grain more than the present. The standard of weight proposed, is that of a cubic tenth of a foot, (or the thousandth part of a cubic foot) of rain water, to determine and constitute the ounce.

Mr. Jefferson's second plan—(supposing a more thorough reformation is contemplated,) advocates a decimal division of all measures. "The unit of [linear] measure is still that which must give law through the whole system; and from whatever unit we set out, the coincidences between the old and new ratios will be rare. All that can be done will be to choose such a unit as will produce the most of these. In this respect, the seconds' rod has been found on trial to be far preferable to the seconds' pendulum."

Assuming, then, Leslie's seconds' rod of 58.72368 inches as his standard, and dividing it into five equal parts, Mr. Jefferson takes this fifth part, or 11.744736 inches as the length of the new "foot." From this is derived the following table:—

Decimal Table of Long Measure.

	1 point	=	.011 in.
10 points make	1 line	=	.117 in.
10 lines "	1 inch	=	1.174 ins.
10 inches "	1 foot	=	11.744736 ins. ($\frac{47}{48}$ foot.)
10 feet "	1 decad	=	9.787 feet.
10 decads "	1 rood	=	97.872 "
10 roods "	1 furlong	=	978.728 " ($1\frac{1}{3}$ furlongs.)
10 furlongs "	1 mile	=	9787.28 " ($1\frac{6}{7}$ miles.)

The proposed measures of capacity are derived from the cube of the new "foot;" this cubic "foot" (containing 1620 cubic inches,) the standard "bushel" — being about one fourth less than the then medium English bushel. The table is as follows: —

Decimal Table of Capacity Measure.

	1 meter	=	1 cubic inch (n. s.) or 1.62 cub. in.
10 meters make	1 demi-pint	=	$2 \times 2 \times 2\frac{1}{2}$ ins. (n. s.) or 16.2 cub. in.
10 demi-pints "	1 pottle	=	$5 \times 5 \times 4$ ins. (n. s.) or 162. cub. in.
10 pottles "	1 bushel	=	$10 \times 10 \times 10$ ins. (n. s.)
10 bushels "	1 quarter	=	9.375 cubic feet.
10 quarters "	1 last	=	93.753 cubic feet.

This table is intended for both liquid and dry measure. The "pottle" is about one sixth more than the English measure of that name. The "quarter" is about one fifth less, and the "last" about one seventh more than the English.

The standard of weight is derived from the preceding measures, by taking one cubic "inch" (n. s.) or one "meter" of rain water, to be called the "ounce" (n. s.) Its weight is $\frac{5}{6}$ oz. Troy, or $\frac{15}{16}$ oz. avoirdupois. The following is the table:—

Decimal Table of Weights.

		1 mite	=	.041 grains Troy.
10 mites	make	1 minim	=	.4101 "
10 minims	"	1 carat	=	4.101 "
10 carats	"	1 double-scruple	=	41.017 "
10 double-scruples	"	1 ounce	=	410.17 "
10 ounces	"	1 pound	=	.5859 lb. av. ($\frac{3}{4}$ lb. tr.)
10 pounds	"	1 stone	=	5.8595 lbs. "
10 stones	"	1 kental	=	58.5957 " "
10 kentals	"	1 hogshead	=	585.9574 " "

When we reflect that the system of metrology here displayed was perfected by Mr. Jefferson before any steps had been taken by the French government toward the decimal re-organization of weights and measures in that country, we must regard it as a memorial in the highest degree creditable to the judgment and contriving skill of its author; and as one of many illustrations of the varied activity of his mind, and of the interest he ever felt in all schemes for human improvement. The great superiority of his proposed scales of measure, to those in common use, cannot be questioned; and their adoption would have been a real and signal public benefit. The tables above presented, form a connected and complete system, each depending directly upon the one preceding, and necessarily flowing out of it, and all determined from a single and invariable natural standard by a very simple and beautiful mode of derivation.

In this respect however, the French system is by far the best of all that have yet been devised. Starting with a carefully measured quadrant of the earth's meridian, and dividing it into 10 million parts, this system presents us with a "metre"* as a universal standard to which all others may be referred. Indeed, if a decimal system of weights and measures is to be ultimately adopted, there appears to be none that has such just claims to our acceptance as that of the French; and although it would be much more difficult of popular introduction than a simple decimalization of our own divisions, as previously indicated, and therefore less "practicable," your Committee does not hesitate to express the opinion that it would be in every way superior, both in regard to the precision of its measures, and the simple and philosophical character of its divisions; besides all which it has the immense advantage of being already introduced and in successful practical operation throughout the great empire of France; and every extension of its use would be an important step in the progress toward a uniform system among all nations.†

* Equal to 39.370788 inches; very nearly the length of the second's pendulum, and not much longer than our yard.

† In the Report made by Prof. A. D. Bache, Superintendent of Weights and Measures, to the Secretary of the Treasury, Dec. 30, 1856, it is stated that the French metrical system "has extended widely beyond the boundaries of France, and has been adopted by law in Spain, Belgium, Greece, Holland, Lombardy, Poland, and Switzerland, in Europe,—and Chili and Mexico, on this continent." (Page 87.)

The scales of length, measure, and weight, with their respective values, may as well be here presented:

French Table of Lengths.

		1 *millimetre*	0.03937 in.
10 *millimetres*	=	1 *centimetre*	0.39370 in.
10 *centimetres*	=	1 *decimetre*	3.93707 ins.
10 *decimetres*	=	1 METRE	1 yd. 3.37079 "
10 METRES	=	1 decametre	10 yds. 2 ft. 9.7078 "
10 decametres	=	1 hectometre	109 yds. 1 ft. 1.0788 "
10 hectometres	=	1 kilometre	1093 yds. 1 ft. 10.788 "
10 kilometres	=	1 myriametre	6 miles 376 yds. 11.88 "

The unit of capacity is derived from the tenth part of the METRE, being the cubic *decimetre*, and is called a LITRE. This gives the following scale of values:

French Table of Measures. (Capacity.)

		1 *millilitre*	16.2318 ♏.
10 *millilitres*	=	1 *centilitre*	2 drs. 42.3180 ♏.
10 *centilitres*	=	1 *decilitre*	3 oz. 3 drs. 3.168 ♏.
10 *decilitres*	=	1 LITRE	2 pts. 1 oz. 6 drs. 31.68 ♏.
10 LITRES	=	1 decalitre	2 galls. 5 pts. 2 oz. 1 dr. 18.336 ♏.
10 decalitres	=	1 hectolitre	26 galls. 3 pts. 5 oz. 5 drs. 3.36 ♏.
10 hectolitres	=	1 kilolitre	264 galls. 1 pt. 8 oz. 2 drs. 33.6 ♏.
10 kilolitres	=	1 myrialitre	2641 galls. 7 pts. 3 oz. 1 dr. 36 ♏.

The unit of weight, is one *millilitre* (a cubic *centimetre*) of distilled water, in a vacuum, at its maximum density, (39° Fahr.) This unit of weight is called the GRAMME, giving the following scale:

French Table of Weights.

		1 *milligramme*	0.0154 grs.
10 *milligrammes*	=	1 *centigramme*	0.1543 grs.
10 *centigrammes*	=	1 *decigramme*	1.5434 grs.
10 *decigrammes*	=	1 GRAMME	15.434 grs.
10 GRAMMES	=	1 decagramme	2 dr. 1 scr. 14.3 grs.
10 decagrammes	=	1 hectogramme	3 oz. 1 dr. 2 scr. 3.4 grs.
10 hectogrammes	=	1 kilogramme	2 lbs. 8 oz. 1 dr. 14. grs.
10 kilogrammes	=	1 myriagramme,	26 lbs. 9 oz. 4 dr. 1 scr.

Beautiful and simple as this system appears, and clear as its nomenclature is to those familiar with the Greek and Latin tongues, it is yet open to animadversion on practical grounds, in that its language is that of the philosopher, and not of the tradesman or the business man. To all but classical scholars — that is, to the large majority of men, — the terms used in all the above tables are difficult and unmeaning; to be acquired and appreciated only by a laborious effort of abstract memory, and even when thus acquired, constantly liable to be confounded and mistaken. Its metres and litres, its myriametres and myrialitres, its decigrammes and decagrammes, are admirably contrived to bewilder the uninitiated, but of all possible devices are the least adapted to the common uses of daily life. To obtain a ready and direct apprehension of the values of different denominations of measure, it is necessary that each should be recognized as an independent unit, without any reference to its fractional or multiple derivation. Thus, "ounces" or "inches" are at once seized upon by the mind as distinctive standards of value; and the fact that these terms both signify "twelfths" (being derived from the Latin "uncia") never enters into our contemplation when using them. The coin a "cent" has come to signify a "one" and not a "hundredth." What is really needed then for the popular service, is a set of names, brief, easy, and distinctive by a wide separation of sound, however arbitrary or unmeaning may be their origin. In this view of the matter, the rude and indefinite vulgarisms of "grains" and "scruples," "feet" and "rods," "gills" and "gallons," are infinitely preferable to the scientific jargon of *centigrammes* and *milligrammes*, and *hectogrammes* and *kilogrammes*. In fact, the French system has totally ignored all units, excepting the single one selected as the standard for each table. Thus in weight, the French cannot be said to have any other measure than the gramme; and instead of resorting to the dead languages for so familiar a thing as a simple numeration table, it would be much better to speak of and write down, the multiples or divisions of this weight as a thousand or a hundred grammes, or as so many hundredths or thousandths of a gramme. This, in plain English (or

plain French) would be understood by every one, and would just as conveniently express every thing that is contained in the high sounding terms we have characterized as "scientific jargon."* Should these measures be therefore introduced, we should strongly urge the entire abolition of the French nomenclature, and the complete naturalization of the different scales by the substitution of more familiar terms from our vernacular tongue. This is precisely what has been done in Holland, which has adopted the French measures entire, since 1817, but has applied to them the old and native designations of the country, — the English apothecaries' weight being, however, still retained. In Belgium, also, the same method has been pursued.

A practical defect in the working of this system which has been demonstrated by experience, is its incapability of binary divisions; a defect which of course attaches equally to every decimal scale; and one which has always strikingly displayed itself wherever this scale has been brought into popular use, either for the estimation of lengths, bulks, weights or values. In our own country the decimal scale has been applied only to the currency, and we find that in spite of the legal division of the *dollar* into tenths, and its seeming establishment by the coinage and circulation of *dimes*, — the people persist in cutting it up into quarters, eighths, sixteenths, and even thirty-seconds, to the utter neglect of the coins actually established by law, and to the inconvenience, confusion, and loss, resulting from the necessary involvement of interminable and unmanageable fractions. And yet in every department of trade, the current prices are derived exclusively from this process of successive halv-

* While thus strongly expressing our objection to the *nomenclature* of the French tables, (whose very fault is its excess of system,) it would be unjust not to acknowledge, and ungenerous not to admire, the catholic sentiment which dictated it. The eminent philosophers to whom belongs the honor of developing a metrology by far the most perfect that has yet been devised, felt as if they were legislating for the civilized world. Desirous that all might have the benefit of their labors, they rejected all the familiar terms employed in France, and naturally resorted to the great storehouse from which the scientific world has ever been accustomed to draw its technical phraseology; exhibiting in this, their anxiety to adopt a language which might be acceptable to all nations. Unfortunately, it is suited to none. The language of science cannot be that of the shop and the market-place.

ing, never from the process of tithing as ordained and defined by law. The modern coinage of the thirty-second of a dollar, — a three-cent piece, a coin "without a name," as it is without a place in any decimal scale, — may be regarded as a feeble and imperfect recognition by law of the insufficiency of our much admired and boasted decimalization of moneys to meet the actual wants and necessities of trade and daily business life.

Many have supposed that this is all a matter of practical indifference, and that it merely requires the decisive sanction of legislative authority to accustom a people to any set of subdivisions. Such an opinion, however, exhibits both a blindness to the lessons of all experience, and an inattention to many of the most important and subtle theoretical considerations affecting the relations of value and our apprehension thereof.

Binal progression may be regarded as preëminently the natural scale of division. This fundamental fact is indeed illustrated in the very origin of the word *division*. The binary scale is in the first place the lowest and simplest of all the geometrical progressions. It is that of which we have the most ready and precise conception; indeed, it may be said to be the only one of which we have any accurate appreciation beyond the second or third term.* It is that by which we most rapidly and nearly approach any vague quantity we may desire to employ; hence its universal use in trade. It is that which in any system of independent units of measure (as in weights, or coins) furnishes us with the means of representing the greatest range of particular values, by the smallest number of pieces. It is that which affords us the easiest practical measure; thus we can fold a string, a sheet of paper, or any other flexible material, or we can cut an apple, or a loaf of bread, at once and with great precision into halves, quarters, and eighths, while we should have to make repeated trials to divide the same into thirds or fifths, and then attain the result only tentatively and approximately. And lastly, it appears to be the most natural of scales,

* Thus, 1, 2, 4, 8, 16, 32, 64, &c., can be readily apprehended as repeated doublings, — while 1, 3, 9, 27, 81, &c., leave the mind confused in the attempt to follow up successive triplings.

from the very common use of the two hands in separating objects into pairs.*

Such being the claims then, of the binary scale of geometrical progression, and such its obvious advantages over all others, it is not surprising that this should be found to be practically the prevalent mode of distributing the more common weights and measures throughout the world, whatever may be the multiples or divisions enacted by law.

The Roman weights, in general use throughout the empire (that is, throughout the civilized world) for some centuries after the Christian era, were, by means of intermediate subdivisions (introduced by the common consent of traders) practically distributed upon a binary scale. The smallest weight was the *lens* or *primus* (equal to about three quarters of a Troy grain) of which 18 made one *quadrans*, (the quarter drachm); 2 of these made 1 *dimidium*, (half-drachm); 2 of these made 1 *drachma*, (very nearly our drachm); 2 drachmæ made 1 *sicilicus* or *siclus;* 2 of these made 1 *semiuncia* or *assarius;* and 2 of these made 1 *uncia*, the twelfth part of the *libra* or pound. So with the divisions in universal use at the present day*;* we find that a nest of avoirdupois weights comprises ¼ oz., ½ oz., 1 oz., 2 oz., 4 oz., 8 oz., and 16 oz., or 1 pound, and sometimes a 2 pound weight, and a 4 pound weight*;* and by this scale of binal progression or division, almost everything is purchased at retail.† Our yardsticks are found to be

* "The classification by pairs which nature points out would suggest the simplest mode of reckoning. Counting these pairs again by two, and repeating the procedure, we arrive by progressive steps at the radical terms, 4, 8, 16, &c." (*Edinburgh Review*, for May, 1811, vol. 18, page 185.)

The celebrated Leibnitz, so eminent as a mathematician as well as a philosopher, struck with the simplicity and peculiar capabilities of this scale, proposed and strongly urged the introduction of Binary Arithmetic. He showed that the Binary system, in addition to its extreme facility, possessed peculiar value in discovering the properties of numbers, and in constructing tables, &c. He did not, however, recommend it for general use, from the increased number of figures required to express ordinary amounts.

† Mr. Adams in his Report, speaking of the old and new measures of capacity in France, remarks:—"The most easy and natural divisions of liquids are in continual halvings; and the Paris pint was thus divided into halves, quarters, eighths, sixteenths, and thirty-second parts, by the name of chopines, half-setiers, possons, half-possons, and roquilles. The half-setier, just equivalent to our half-pint, was the measure in most common use for supplying the daily necessities of the poor; and thus the decimal divisions of the law became snares to the honesty of the seller, and cheats upon the wants of the buyer." (*Report on Weights and Measures.*)

divided not into the legal feet and inches, but into halves, quarters, eighths and sixteenths. Precisely so with the inch, which is never divided into its primitive "three barley corns," but almost always, like the yard, by the binal scale, into eighths and sixteenths, though occasionally divided for particular purposes into twelfths, or into tenths. The operation of this great law is quite as strikingly exhibited in France, where the popular necessities have compelled the introduction of binal divisions, not recognized by the established decimal scales, nor, indeed, strictly compatible therewith.*

Mr. Peacock, in his admirable treatise on "Arithmetic," in the *Encyclopedia Metropolitana*, thus sums up his review of the French system. "The decimal subdivision of these measures possessed many advantages on the score of uniformity, and was calculated to simplify, in a very extraordinary degree, the arithmetic of concrete quantities. It was attended, however, by the sacrifice of all the practical advantages which attend subdivisions by a scale admitting of more than one bisection, which was the

* "The supreme law of the land was made to bend before the will of the people, and after a vacillating legislation, an imperial decree was issued in 1812, which while it retained the *units* of the new weights and measures, allowed them to be divided by other than decimal numbers. Thus gradually were the old names reintroduced and applied to measures and weights to which they never before belonged. For retail sales of all articles which are sold by weight, the shopmen were allowed to employ the following usual weights. The pound (livre) equal to half a kilogramme, or 500 grammes, which shall be divided into 16 ounces. The ounce (once) or one-sixteenth part of the pound, which shall be divided into eight gros. The gros, or one-eighth part of the ounce shall be divided into halves, quarters and eighths. They shall bear with their appropriate names the indication of their weight in grammes." (*Journal of Pharmacy for October*, 1830, vol. 2. Page 201. Article on "Weights and Measures," by Dr. B. Ellis.)

This decree of 1812, was, however, in 1840 revoked; and the "Systeme Usuel" has since then been interdicted by law, under penalties; the decimal metrology being alone permitted, and strictly enforced. The practical working of the system, however, during the last twenty years, has but furnished another illustration to the oft-cited apophthegm of Horace:

"Naturam expellas furca, tamen usque recurrit."

Binary divisions, as we are informed, are still common in trade, either in the open guise of fractions, or in the form of higher multiples of lower units. On turning over a file of the *Moniteur Universel* (a daily paper of Paris,) for the year 1852, we find the prices in the money market constantly interpolated with quarters and eighths. Thus in the number for Dec. 30th, 1852, we have in the list of prices on the last page, quotations of $98\frac{7}{8}$ francs; $98\frac{3}{4}$ francs; $43\frac{3}{8}$ francs; &c.

case with those previously in use; *and it may well be doubted whether the loss in this respect was not more than a compensation for every other gain.*" Such a statement from one who has given the whole subject so careful a study, certainly deserves our most serious consideration.

The masterly and comprehensive report on the subject of weights and measures, made to Congress in 1821, by Mr. Adams, when Secretary of State, contains the following judgment: "The experience of France has proved that binary, ternary, duodecimal, and sexagesimal divisions are as necessary to the practical use of weights and measures, as the decimal divisions are convenient for calculations resulting from them; and that no plan for introducing the latter, can dispense with the continued use of the former. . . . . From the verdict of experience, therefore, it is doubtful whether the advantage to be obtained by any attempt to apply decimal arithmetic to weights and measures, would ever compensate for the increase of diversity which is the unavoidable consequence of change. Nature has no partialities for the number ten; and the attempt to shackle her freedom with them will for ever prove abortive."

So in the interesting paper of Dr. Ellis, (in the *American Journal of Pharmacy*, vol. 2, page 202,) the French decimal system is thus referred to: "Every one is struck, at the first glance of this system, with the beautiful simplicity which it derives from decimal arithmetic. It appears, however, to have been overlooked, that, although decimal arithmetic is admirably designed to facilitate the calculation of mere number, it is not equally well suited to the divisions of material things."

Much to the same effect has been the result of the commission appointed lately in England to consider the subject of a decimal coinage. The commissioners, after a full discussion and investigation of the subject, have very recently reported against any change; their report being drawn up in the form of a series of twelve resolutions. The seventh resolution is as follows: "That as regards the comparative convenience of our present coinage, and of the pound and mill scheme, for the reckonings of the shop

and the market, and for mental calculations generally, the superiority rests with the present system, in consequence, principally, of the more convenient divisibility of 4, 12 and 20, as compared with 10, and the facility for a successive division by 2, that is, for repeated halving, in correspondence with the natural and necessary tendency to this mode of subdividing all material things; and with the prevalence of binary steps in the division of our weights and measures."

In the view, then, of this pervading law or principle of all human metrology, so well established, and so distinctly recognized, it becomes an obvious necessity, in adopting a decimal scale, to engraft upon it, the divisions of halves and quarters, at least, (and in the case of the more commonly employed units,—of eighths,) if we would adapt it to the demands of the people, or if we would hope for its permanent establishment. It is true that this would involve a considerable number of subordinate divisions between one denomination of measure and the next below it, as it would be requisite to have separate and distinctive weights, for instance, for the unit, (whatever it might be,) for one and a quarter of the unit; for two, for two and a half, and for five; and it is also true, that the fractional values thus introduced would not be directly referable to the ordinary computations of decimal arithmetic,—thus adding, somewhat, to the complexity and trouble of otherwise very simple calculations; but this is a fault, not of the binary divisions themselves, but resulting from a radical and incurable defect in the decimal system. So long as we continue to count, to add, subtract, multiply, and divide by tens, so long must we submit to this inconvenience, (undoubtedly a serious one,) or we must choose the greater evil of abandoning all attempts at uniformity and consistency of system, and continue, as heretofore, to measure and to weigh by heterogeneous tables, while we perform the necessary operations of comparing, compounding, and distributing these values, by a method or ratio entirely dissimilar; entailing upon ourselves the waste of time, labor and patience, consequent upon a petty scheme of eternal and superfluous reductions.*

* "Perhaps it may be found by more protracted and multiplied experience, that this is the only 'uniformity' attainable by a system of weights and measures for universal

This horn of the dilemma is that which has been accepted by the coinage commission of England, to which a reference has just been made. The eleventh resolution of the Commissioners' Report, is: "That the advantages in calculation and account-keeping, anticipated from a decimal coinage, may, to a great extent, be obtained without any disturbance of our present coinage, by a more extensive adoption of the practice now in use at the National Debt Office, and in the principal assurance offices; viz., of reducing money to decimals, performing the required calculations in decimals, and then restoring the result to the present notation." With our experience of a decimal coinage, (notwithstanding its imperfections,) this is not the horn likely to be selected by Americans in attempting a reform in weights and measures.

An expedient has been suggested by some, for facilitating division in decimal notation, which is ingenious, and deserves a notice. The project is to adopt a uniformly decimal system of weights and measures, but to estimate entirely by "cents,"—by simply suppressing every alternate denomination; thus, while reckoning decimally, we should traffic only centesimally. Our practical application of this method in all our money transactions, in which dimes are entirely suppressed in the market, (though still having their place in the columns of the ledger,) and our estimates made in *dollars* and *cents*, familiarizes our minds to the process, and enables us to see how such a system might be indefinitely extended, by the simple device of counting by double places of figures. The French table of weights would stand thus:

100 deci-milligrammes	make	1 centigramme.
100 centigrammes	"	1 gramme.
100 grammes	"	1 hectogramme.
100 hectogrammes	"	1 myriagramme.

This suppression of the alternate denominations would have the advantage of abolishing the very objectionable terms *decigramme*

use: that the same material instruments shall be divisible decimally for calculations and accounts; but in any other manner suited to convenience in the shops and markets; that their appropriate legal denominations shall be used for computation, and the trivial names for actual weight or mensuration." (*Adams's Report.*)

and *decagramme*. Instead of the extreme awkwardness of taking one quarter of a gramme ($2\frac{1}{2}$ decigrammes,) we are furnished with the value in whole units, by taking twenty-five centigrammes.

Simple and taking as this proposal is, it is, in our opinion, not free from serious objections. It, in fact, complicates rather than simplifies, by giving a very wide range for estimating values. While it thus multiplies the units, and enlarges the interval between them tenfold, — it only furnishes us with a *single* additional bisection, namely, the quartering. An eighth would still require a fractional expression. Its benefit, therefore, bears no proportion to the increased trouble and confusion involved. The necessity universally felt for quarternal and octaval divisions, would infallibly operate here as it has in our currency; and we should constantly hear of $37\frac{1}{2}$ hundredths of a pound; $62\frac{1}{2}$ hundredths of a pint; &c., which would be, in no respect, better than $3\frac{3}{4}$, or $6\frac{1}{4}$ tenths. The truth is, we need more frequent denominations than decimal ones, rather than more distant stepping-stones; and for some purposes, even the binary ratio of progression is not too slow. In looking over the various tables of weight and measure prevailing throughout Europe, it will be found that a large majority of the factors are 2, 4, and 8, with occasional resort to 3 and 6, — the number 4 being, perhaps, the favorite number for the more customary denominations.*

In the course of this investigation, the Committee has been constantly impressed with the inseparable nature of the various branches of a metrical system; and with a consequent conviction of the impossibility of giving that exclusive and confined attention to the weights and measures of our own particular profession, which was probably intended in the appointment of the commission. We have felt, more and more, the necessity of a conformity among all the measures of trade and commerce, and of the reference of all to common laws, and to a single standard. We find that many of the evils that are experienced by us, are those pre-

* This is rendered very apparent on turning over the pages of Woolhouse's little work on the "Weights and Measures of all Nations." No. 101, of Weale's Rudimentary Series. Time does not permit the reference to examples.

vailing in all departments; that no improvements or reforms can be either efficient or enduring, which do not look to the welfare of the whole; and that the agitation of the subject carries with it consequences of far wider scope, and of more momentous interest than most of us had probably conceived. The empiric may be satisfied to alleviate, temporarily, some prominent or more distressing symptom; it is the province of the scientific practitioner to search into the origin of the malady, and to so administer to the real sources of the grievance, as may effect a thorough cure. We doubt, therefore, whether we are at present in a condition to do more than indicate what we regard as the most desirable objects and directions of reform; contributing our share of labor and consideration toward perfecting the great and wide-spread project of improvement, and leaving to those who can most efficiently act in the matter, to further or to better our suggestions.

Entertaining this feeling upon the subject, we are not prepared to fully approve the recent action of the British Colleges engaged upon the revision of the Pharmacopœia, in changing merely the scale by which medicinal preparations are to be dispensed. Discarding the Troy weights, and dividing the avoirdupois pound into 7680 parts, to be called grains, they have presented us with the following table:

British Pharmaceutical Table of Weights.

	1 grain,	or	1 gr.	=	0.911458 Troy gr.
20 grains make	1 scruple,	"	20 grs.	=	0.911 scruple.
3 scruples "	1 drachm,	"	60 "	=	2 scr. 14.6875 grs.
8 drachms "	1 ounce,	"	480 "	=	7 drs. 0 scr. 17½ "
16 ounces "	1 pound	"	7680 "	=	1 lb. 2 oz. 4 dr. 2 scr.

While we recognize this scheme of division as a decided improvement upon the present anomalous Troy scale, we do not think it worth the sacrifice involved, nor do we see in it the qualities likely to give it a permanency of continuance, or a generality of acceptance. The Troy grain is a standard so widely used, and in medicine especially, is one of so great value, as the exponent of so much knowledge and experience, that it should not be lightly set

aside; and before we incur the greatly increased labor, inconvenience, and confusion, necessarily attendant upon the transition, we should well consider whether we have attained all the benefit within our reach, or whether, at no greater cost, we might not reap the advantages of a far more perfect system.

Amid the conflicting claims of the numerous plans proposed for simplifying and uniting our incongruous metrology (of which only a few of the leading ones have been adverted to,) there appears, at first sight, so much of irreconcilable contrariety, that it might be concluded that a combination of the respective advantages contemplated was hopeless and impossible; and that we were only left to a choice of evils. A more careful scrutiny will, however, as we believe, discover a philosophy in these very discrepancies, and furnish us with the elements of a practicable concord. On the one side, the convenience of a system of divisions or multiples conforming exactly to that by which we are compelled to perform all arithmetical operations, is so obvious, and so universally recognized,* that the advocates of an entire decimalization are certainly justified in their zeal. On the other hand, the necessity of binal progression and division, though not so generally acknowledged, is by all who have given the subject a careful study, so fully appreciated, as being, at least, as fundamental as that of the decimal scale, that those who urge the retention of all such denominations as are measured by the powers of 2, are no less justified. Which policy must, then, be sacrificed?

"The elementary principle of decimal arithmetic," says Mr. Adams, "is supplied by nature to man within himself, in the number of his fingers. Whatever standard of linear measure he may assume in order to measure the surface or the solid, it will be natural to him to stop in the process of addition, when he has counted the tale equal to that of his fingers. . . . . But while decimal arithmetic, thus for the purposes of computation, shoots spon-

* "The great improvement of having but one arithmetical scale for reckoning integers and fractions of every kind, ... is one so obvious, and, withal, so little difficult, that it is a matter of surprise that it should not have been attempted till near a thousand years after decimal arithmetic itself was first introduced into Europe." (*Edinburgh Review, for January*, 1807, volume 9, page 373.)

taneously from the nature of man and of things, it is not equally adapted to the numeration, the multiplication, or the division of material substances, either in his own person, or in external nature. The proportions of the human body, and of its members, are in other than decimal numbers. The first unit of measures for the use of the hand, is the cubit, or extent from the tip of the elbow to the end of the middle finger; the motives for choosing which are, that it presents more definite terminations at both ends, than any of the other superior limbs, and gives a measure easily handled and carried about the person. By doubling this measure, is given the *ell*, or arm, including the hand and half the width of the body, to the middle of the breast; and by doubling that, the *fathom*, or extent from the extremity of one middle finger to that of the other, with expanded arms,—an exact equivalent to the stature of man, or extension from the crown of the head to the sole of the foot. For subdivisions, and smaller measures, the *span* is found equal to half the cubit, the *palm* to one-third of the span, and the *finger* to one-fourth of the palm. The *cubit* is thus for the mensuration of matter, naturally divided into 24 equal parts, with subdivisions of which 2, 3, and 4, are the factors; while for the mensuration of distance, the foot will be found equal to one-fifth of the pace, and one-sixth of the fathom." (*Report to the Senate and House of Representatives, Feb.* 22, 1821, *on Weights and Measures.*)

"The fingers," says Dr. Lardner, "were naturally the first objects which presented to the mind the idea of number; and they furnished, also, a set of natural counters by which the number of things might be marked and expressed. The fingers, being continually in view, familiarized the mind with the contemplation of every number of objects not exceeding ten. It was natural, therefore, that ten should be adopted as the number of objects to form the first group. . . . . . Although ten has been so generally adopted as the *radix* of systems of numeration, as to leave no doubt of its origin, yet it is not the only one which has been used, nor is it the only radix having a natural origin. The fingers of one hand rendered the number five familiar to the mind, before the conception of ten as a distinct number presented itself. It

was even more natural and obvious, that the fingers should be contemplated as two groups of five, than as a single group of ten." (*Treatise on Arithmetic,* book i, chap. 1, p. 5-6.)

The gradual and successive development of these scales, is so well set forth in Mr. Peacock's valuable treatise, that we think no apology is necessary for a somewhat lengthened extract from it, even at the cost of some repetition.

"The decimal scale of numeration is not the only one which may be properly characterized as a natural scale. In numbering with the fingers we might, very naturally, pause at the completion of the fingers on one hand; and registering this result by a counter, or by any other means, we might proceed over the fingers of the same hand again, or with the fingers of the second hand, and register the result by another counter, or replace the former by a new counter which should become the representative of ten. . . . . Again, the scale of numeration by twenties has its foundation in nature, equally with the quinary and denary scales. In a rude state of society, before the discovery of other methods of numeration, men might avail themselves, for this purpose, not merely of the fingers on the hands, but likewise of the toes of the naked feet; such a practice would naturally lead to the formation of a *vicenary* scale of numeration, to which the denary, or the denary with the quinary, or the quinary alone, might be subordinate. . . . . Of other systems of numeration, the binary might be considered as natural, from the use of the two hands in separating objects into pairs, and from the prevalence of binary combinations in the members of the human body; but the scale of its superior units increases too slowly to embrace within moderate limits the numbers which are required for the ordinary wants of life, even in the infancy of society. . . . . As the necessity of numeration is one of the earliest and most urgent of those wants which are not essential to the support and protection of life, we might naturally expect that the discovery of expedients for that purpose should precede the epoch of civilization, and the full development and fixing of language. That such has been the case, we shall find very fully and clearly established,

by an examination of the numerical words of different languages; for, without any exception which can be well authenticated, they have been formed upon regular principles, having reference to some one of those three systems which we have characterized as natural: the quinary scale, whenever any traces of it appear, being generally subordinate to the denary, and in some cases, both the quinary and denary scales being subordinate to the vicenary. In some cases, also, we shall find, from an examination of primitive numerical words conveying traces of obsolete methods of numeration, that the quinary, and even the vicenary scales have been superseded altogether by the denary." (*Encyclopedia Metropolitana*, art. "Arithmetic," vol. I, p. 371.)

Decimal arithmetic thus appears to be coëval and coëxtensive with the human race. It is, indeed, perhaps, the most universal of human institutions, — at least as universal as language itself.* From this universality, most writers have called it the "natural" system; but on examining the question whether the number *ten* possesses any intrinsic excellence or convenience to recommend it, — any peculiar fitness as a ratio of geometrical progression, we find but one answer, — it has none. It differs from any other number only in quantity, not in quality. So far from its presenting any merit or advantage over its compeers, it is almost the last number which a true science of arithmetic would have selected for the important function of a radix of numeration. Its universality flows simply from the fact that the necessities of man impelled a selection, in the very earliest infancy of the race, long before the invention of letters, and while yet a language was but slowly being formed; and the selection comes to us stamped with the crude impress of a most irrelevant accident! Had the six-fingered giant slain by Jonathan, (2 Sam. xxi. 20,) lived early enough to be the father of the first unreasoning tribes, we should have had a duo-

* "We are not aware that there is an instance extant in any language, ancient or modern, of a complete numeral system formed upon a single radix other than the decimal. We have abundant examples, as already stated, of the occasional appearance of the quinary and vigesimal radices in scales which are chiefly decimal; and examples may be produced from numerals of the Celtic dialects of a strange mixture of the quinary, denary, and vicenary systems."—(*Lardner's Arithmetic*, chap. 1, p. 17.)

decimal arithmetic; or if, like the fowls of the air, we had usually but four toes to our extremities, we should now have been able to calculate only octavally; and in either event we should have been much more skilful computers than we are at present.*

Decimal numeration is "natural" then, only in the sense that *ignorance* is natural. The fingers have no more real or "natural" relation to the properties of number, than have any other organs or divisions of the human body; and mathematically or philosophically considered, the *digit* is, therefore, no more a typical *unit* than a tooth, (of which there are thirty-two,) or the leg of a spider, (of which there are eight,) or the petal of a flower, (of which there may be any number.) Nor have any but the most ignorant races — those without a literature and an alphabet, — ever occasion to group and tally by their fingers. Only from unlettered savages could such a scale, therefore, have been derived.

It has been a favorite theory with a certain class of thinkers, that primitive man was a highly civilized being, — "a scholar and a gentleman;" and that the decay of states, and the decline of civilizations so unfortunately frequent in his history, but manifest his prevailing tendency to degeneration. Our universal arithmetic furnishes us with one of the most striking refutations of such a fancy. Wherever over the broad earth, the decimal scale exists, there have we the enduring monument of the ancestral savage — counting by his fingers or his naked toes.† Had any intelligent fore-

* "There can be no doubt that if man had been a twelve-fingered animal, we should now possess a more perfect system of numeration than we do. Whatever be the radix of the scale, it would always be a convenience to be able to subdivide it with facility, without resorting to the more refined expedient of fractional language."—(*Lardner's Arithmetic*, chap. 1, p. 21.)

† The German word for ten — *zehen*, — signifies "toes," being the plural of the word *zehe*. We do not generally or readily recognize this intellectual association in our own language; and yet the Saxon word — *ta* — a "toe," is in the plural *tan*. The *daktul* (δακτυλος) of the Greeks, and the *digit* (digitus) of the Romans, which signified either "finger" or "toe," appear evidently affiliated to the *deka* (δεκα) of the one and the *decem* of the other; although the genealogy (as in English) was probably more ancient than the languages themselves. So uniform are the laws of mind and matter, that we have only to select some rude and isolated tribe of modern savages, to discover with a naturalist's confidence, the exact process of development in numeration, with the aborigines of our race, millenniums on millenniums ago. Klaproth, in speaking of the inhabitants of the peninsula of Kamtschatka, says: "It is very amusing to see them attempt to reckon above ten; for having reckoned the fingers of both hands, they clasp them together, which signifies ten; they then begin at their toes and count to twenty; after which they are quite confounded, and cry "matcha," that is, where shall I take more?" — (*Sprachatlas*, page 16.)

thought ever presided over the inception of a numerical scale,— had any comprehensive conception of the uses and purposes of figures, in any single instance guided the selection of a ratio for their multiplication,— that ratio must inevitably have been something else than *ten;* the duplication of an odd number,— incapable of any other division,— neither a square, a cube, nor other power of any integer,— and in its successions among the most inefficient for the expression of fractional values, or for the extraction of roots. And if among the patriarchs of the human family, a rational scale had ever been so devised, some traces of this wiser system must have been found, to give a "sign" and memento of man's pristine elevation.

"The number ten," remarks Mr. Anderson, in his treatise on Arithmetic, "has been adopted by every civilized nation for the radix of the numerical scale. It has no peculiar advantages to recommend it, and seems to have been selected for that important function, merely because it expresses the number of the human fingers. We must regret that a circumstance so totally unconnected with every scientific consideration, should have determined an elemental principle, of the last importance to one of the most abstract, as well as one of the most useful of all the sciences; and that the decimal notation should still be retained, notwithstanding its evident imperfections, and the superior claims of other scales." (*Edinburgh Encyclopedia;* edited by Sir David Brewster, art. "Arithmetic," vol. 2, page 411.)

An able and philosophical writer in the *Edinburgh Review* holds very similar language. "Ten has indeed," he observes, "no advantage as the radix of numerical computation; and has been raised to the dignity which it now holds, merely by the circumstance of its expressing the number of a man's fingers. They who regard science as the creature of pure reason, must feel somewhat indignant that a consideration so foreign and mechanical, should have determined the form and order of one of the most intellectual and abstract of all the sciences." (*Edinburgh Review*, for January, 1807, vol. 9, page 376.)

A large number (perhaps even a large majority) of the well-

educated, have been accustomed to regard the decimal system as possessing a peculiar beauty and expressiveness, from the great facility with which the ordinary operations of arithmetic are performed by it. Indeed, after laboring at the tedious and troublesome reductions of compound numbers (consequent upon other scales of progression) unfortunately so often required to be made, the relief of a simple addition or multiplication in the homogeneous units of our common scale, is too striking not to excite a feeling of admiration for the easier process. It appears not to be generally considered, however, that this facility of computation is in no respect due to the series of "tens" by which we count, but is derived exclusively from the admirable notation in which the series has been clothed, and through which alone, we are in modern times made acquainted with it; and from the perfect conformity of the notation to the series. Any other scale will be found to exhibit an equal facility, if the same notation be employed, and made to correspond strictly with the selected scale. If, like the old Arabian philosophers, or like the ancient Greeks and Romans, we were compelled to calculate by a set of *alphabetic* numerals, we should be able to better realize how much we are indebted to that simple and yet grand invention of India, the "cypher-figures," or the set of figures with the device of local value.* This system of numerical language presents us with a formula of geometrical progressions, so illimitable in range, and yet so perfect in its conciseness and distinctness, that it transcends all conception that the ingenuity of man in all coming time shall ever be able to improve it.†

Though from a remote antiquity familiar to the Hindoos, (that wonderful people from whom the civilized world has derived so much,) it was wholly unknown to the nations of the earth until

* See note A — appended to this Report.

† "This method which is now used by every civilized nation, has all the precision we can desire, with the important advantages of conciseness and simplicity. It is impossible indeed for human ingenuity to contrive anything better adapted to the purposes of calculation. A more convenient scale than the decimal, might be substituted; but the principles of the notation itself, are incapable of improvement." (*Edinburgh Encyclopedia*, article "Arithmetic.")

comparatively modern times; having been first introduced into Arabia, less than a thousand years ago, and from thence by slow and successive centuries into the various languages of Europe.

However much the Arabic philosopher to whom belongs the honor of having first transplanted the Sanscrit Arithmetic into his own country, may have been impressed with its great power and beauty, he could hardly have appreciated to its full extent, the importance and magnitude of the gift he was instrumental in presenting to the civilized world; a transfer which Sir John Bowring in his "Decimal System," (chap. 2, p. 22,) has characterized as "the greatest step ever made towards the introduction of a universal language among the nations of the world." The Hindoo numerals, from the channel of their introduction into Europe, were generally called the "Arabic figures"—a title they still commonly retain, though it is one hardly just to the people with whom these figures had their origin.

Now although this Hindoo notation has never been popularly applied to any other than the decimal scale, it is obviously a formula of universal applicability; and if made use of to express a system of figures with any other radix than *ten*, would give the same facility to all the calculations performed by that system. Abstracting, for a moment, all specific value from the terms "units," "tens," "hundreds," and "thousands," and regarding them merely as symbols of *local value* (designating only the orders of units,) we may exhibit in a tabular form, a series of scales, with the successive increments of value for each place, according to the radix, or ratio of geometrical progression selected. The letter "U" in the top line, denotes a "unit;" that is, any figure which may occupy a single place.

Table of Arithmetical Scales.

HINDOO NOTATION.	U.	UO.	UOO.	UOOO.			
Scale.	**Units.**	**Tens.**	**Hund's.**	**Thous'ds.**	**Radix.**	**Logarithm.**	**Places.**
Binary.	1	2 —	4 —	8 —	2	.301	$\frac{10}{3}$
Ternary.	1 — 2	3 —	9 —	27 —	3	.477	$\frac{25}{12}$
Quaternary.	1 — 3	4 —	16 —	64 —	4	.602	$\frac{5}{3}$
Quinary.	1 — 4	5 —	25 —	125 —	5	.699	$\frac{10}{7}$
Senary.	1 — 5	6 —	36 —	216 —	6	.778	$\frac{50}{39}$
Septenary.	1 — 6	7 —	49 —	343 —	7	.845	$\frac{20}{17}$
Octonary.	1 — 7	8 —	64 —	512 —	8	.903	$\frac{10}{9}$
Novenary.	1 — 8	9 —	81 —	729 —	9	.954	$\frac{20}{19}$
Denary.	1 — 9	10 —	100 —	1000 —	10	1.000	1
Duodenary.	1 — 11	12 —	144 —	1728 —	12	1.079	$\frac{25}{27}$
Quaterdenary. . . .	1 — 13	14 —	196 —	2744 —	14	1.146	$\frac{50}{57}$
Senidenary.	1 — 15	16 —	256 —	4096 —	16	1.204	$\frac{5}{6}$
Octonidenary. . . .	1 — 17	18 —	324 —	5832 —	18	1.255	$\frac{4}{5}$
Vicenary.	1 — 19	20 —	400 —	8000 —	20	1.301	$\frac{10}{13}$
Tricenary	1 — 29	30 —	900 —	27000 —	30	1.477	$\frac{25}{37}$
Quadragenary . . .	1 — 39	40 —	1600 —	64000 —	40	1.602	$\frac{5}{8}$
Quinquagenary. . .	1 — 49	50 —	2500 —	125000 —	50	1.699	$\frac{10}{17}$
Sexagenary.	1 — 59	60 —	3600 —	216000 —	60	1.778	$\frac{50}{89}$

The number of places for each scale, is inversely as the logarithm of the Radix.

The most striking feature displayed by such a comparison of the different scales is the rapid increase of value in the higher ratios, as compared with the lower. While the *ternary* scale, for example, requires four figures to express so small a number as 27, the *tricenary* scale expresses one thousand times as many, by the use of no more places. The very first inquiry would, therefore, naturally be (in the absence of any other consideration) which would be found more convenient, — a very small radix, or a very large one?

The first and lowest scale of the series — the *binary*, presents with some disadvantages, many very remarkable advantages. In the first place it requires but a single figure, 1, (together with

the cipher for determining its place) to express with facility and precision all the values within the reach of figures.* According to the law of the Hindoo notation, by which every zero multiplies all the value that precedes it, by the amount of the radix, it results that the addition of a cipher to the figure 1, would of course multiply it by two, (instead of by ten as in our common system) — the addition of 2 ciphers, — by two times two, or four, (instead of by a hundred) — the addition of 3 ciphers, by eight; of 4 ciphers, by sixteen; of 5 ciphers, by thirty-two; &c. The first fifteen numbers would read thus: 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111.† The present year, 1859, would require eleven places of figures to express it; namely, 11101000011. Fifty places of figures (or 1 and 49 ciphers,) in the binary system, would require but fifteen places of figures in the decimal system. One hundred places of the binary, (1 and 99 ciphers) would require thirty places of the decimal. So that the former system would involve, on an average, the constant employment of about three and a third times more figures in all our arithmetical operations, than the latter system, or that in common use. This increased expenditure of time and manual labor, would evidently be a very serious inconvenience. On the other hand it must be considered that the writing down of any given mass of figures, in only two characters (always either a 1, or a cipher,) would be much more easy and expeditious than if the mass consisted of ten different characters; so that the actual increase of trouble should be set down at probably not more than *double* that we have at present. This much quantitatively. But in the quality of the work done, the difference will be found immensely in favor of the binary scheme. In the first place no tables would be required to be committed to, and retained by, the memory; either of addition, of subtraction, of division or of multiplication; not even

* It was in reference to this curious property of the scale, that a medal struck in honor of Leibnitz, and to commemorate his invention of the binary system, bore on its reverse, the striking inscription: "Omnibus ex nihilo ducendis sufficit Unum." Unity being very commonly regarded as the symbol of the Deity. — (*Peacock's Arithmetic*, Encyclopedia Metropolitana, vol. 2, page 392.)

† See note B — appended to this Report.

the fundamental "twice two make four!" Every form of calculation would be resolved into simple numeration and notation. In fact, calculation as an effort of mathematical thought, might be said to be entirely dispensed with, and the labor of the brain to be all transferred to the eye and the hand. A perfect familiarity with the notation of the scale, and with the simple rules of position, would enable the operator to determine in every case by mere inspection whether the next figure should be a 1, or a 0. It follows that the only errors possible in such a work, would be the merely clerical ones of the eye or hand; and when we reflect that a large majority of the arithmetical errors committed, are usually those of the brain, fatigued or bewildered by the constant strain upon the attention and memory, this consideration of the increased accuracy of such a system, is one of the very first importance in estimating its value. To many, the relief it proffers in exchanging head-work for hand-work will appear no trifling recommendation; and it may well be doubted, whether in all important and lengthy calculations, the binary system would not be found to afford a real economy of labor, instead of an increase as has been generally supposed.

It has been previously noticed, that the great Leibnitz, the rival of Newton in the invention of the "Differential Calculus," proposed this system, and zealously urged its adoption; although he thought that for more common purposes it would be found too prolix. "De Lagny took the trouble of constructing logarithms on the principles of this arithmetic, as being more natural than those usually employed. . . . . He even proposed to substitute the binary arithmetic for logarithms, affirming that it was more simple and expeditious, and conducted to the object in view in a less indirect manner." (*Anderson's Article on Arithmetic, in Brewster's Edinburgh Encyclopedia*, vol. 2, pp. 376 and 409.) The same writer adds that "Dangicourt has applied the binary notation with greater success to progressions, and proved that the laws of a series may be detected by it more easily than by any other scale." (Page 409.) This results, it may be as well to state, from the fact

that "circulating periods" of figures return far more frequently in this scale than in any other.*

The *Ternary* scale, although it is also a very simple scale, has nothing else to recommend it; being incapable of integral bisection, and having very nearly the redundancy of the binary scale, without one of its advantages. It may be regarded as the most objectionable of all the scales; and indeed none of the odd numbers, could, for a moment, be accepted as a suitable radix of notation.

The *Quaternary* scale, as derived from the second power of the binary scale, has many of its excellences. While it employs less than half the number of digits, of the common or denary scale, to task the memory and attention, it requires only about five places of figures, for three of the latter. It combines, therefore, great simplicity of structure, with a moderate range of notation, and would form a very convenient and practicable system of numeration; while it would furnish an admirable scale of division for weights and measures of all kinds. It is said by Balbi, that a very low and ignorant tribe of Indians in South America, — the Guranos — had names only for four digits, and that after counting these a second time (to eight) they were unable to proceed any further. The correctness of this account, appears, however, to be exceedingly doubtful. It is remarkable, too, that Aristotle mentions a tribe of

* From the comparatively slow rate of progression in this scale, its places of figures advancing in value by the successive powers of *two* only, instead of by the powers of *ten*, as in our present system, it might be supposed that its capability of expressing large amounts was very limited; and there are probably very few who would not be startled by the announcement, that the number of the stars of heaven, including all those made visible by the most powerful telescopes, — the milky way, — and all the mighty nebulæ resolved, — added to the number of the grains of sand upon the sea-shore, would all be expressed by the binary notation in a single row of figures, not extending across a sheet of common letter paper! And yet this can be very easily shown. Assuming that a cubic inch of sand would contain ten million grains, the number of grains of sand necessary to constitute a globe as large as our earth, would not exceed 659 quintillions: an expression requiring but thirty-three figures. In the binary scale this would require only 109 figures. Our sun is a million times larger than our earth. Supposing that even this huge globe were composed of sand, we should have to add but 20 ciphers to these 109 figures to multiply them by a million, and to express the entire number, in the binary notation.

Thrace as being unable to count beyond four,—a statement equally incredible.

The *Quinary* scale, whose notation would require ten places for seven of the *Denary*, has nothing to recommend it; and yet from the accident of man being afflicted with five fingers, it has generally formed the basis of the scale in common use, and traces of it are to be found in perhaps a majority of the nations of the earth. The numerals of Malay and Java were anciently, for the most part, quinary, in subordination to the vicenary grouping. A trace of this system is also seen among the ancient Greeks, in their word πεμπαζεσθαι; (to count by fives;) as it is among the Romans in their notation of numbers above 5, 15, &c. The Persian term for "five" is *pendja;* and *pentcha* signifies the expanded hand. Among the South Sea Islanders, the inhabitants of New Caledonia and the Hebrides, as well as the barbarous tribes of north-eastern Asia, the quinary scale appears still to prevail. The central tribes of North America show also traces of this digital period; and they are frequent among the innumerable languages of Africa. Thus with the Jalloffs, the word for "five"—*juorom*—signifies the "hand." So with the Foulahs, the Jallonkas, the Fellups, &c. There are no examples, however, of the number five ever having been used as a true radix of notation; that is, as a direct ratio of continued progression; 5;—5×5, (25;)—$5 \times 5 \times 5$, (125;) &c. The quinary scale has seldom gone further than 20.

The *Senary* scale would require about 17 places of figures for 13 of the common scale; and its notation would therefore have about a one-third greater extent. Though not one of the most desirable scales, it would be much superior to the denary system. The simplification arising from the reduction of its digits, would much more than counterbalance the extension consequent on the increase of its places. Like the denal scale, it admits of but one bisection; but it possesses the great superiority of admitting at the same time of a trisection. No examples of this scale are to be met with; although it is said to have been at one time decreed in China, by the caprice of an Emperor, who had conceived some astrologic fancy for the number six.

The *Octonary* scale approaches very nearly to the common scale in its capability of expression, as it requires on an average but one ninth more places of figures to represent any given amount; that is, ten places of this scale would be equivalent to nine places of the denary. Being derived from the third power of the binary scale, it possesses most of the advantages of that system; though not its admirable simplicity. Like the quaternary, it admits of continued bisection down to unity; and, of course, of indefinite bisection below 1, by the simple expedient of an inverted, or negative notation, (as in decimal fractions.) As a perfect cube, it has peculiar advantages both as a radix of numeration, and as a ratio of progression or of division for weights and measures; and in the latter respect particularly, there is, perhaps, no other number that would so well express the average range of a convenient metrical multiple.

The *Denary* scale* may be said to present a tolerably convenient mean between the prolixity of a very small radix, and the intricacy of a very large one: besides which, it possesses the immense advantage of a universal establishment. But beyond this, there is nothing to be said in its behalf. Intrinsically, it is one of the most imperfect and troublesome scales which could be selected. Still, the inconveniences of the system should be very serious and very apparent, and the claims of any rival scheme very unquestionable, to justify the advocacy of a change, which would root up all our established forms and habits of calculation, — which would destroy the accumulated products of centuries of industrious thought and toil, — which would entail upon us generations of new labor to attain even the same standard of tabular detail, and statistical information, now possessed; and which, more than all, would wholly demolish, and perhaps hopelessly, that uniformity so essential to

* The name "Decimal," by which our present system of arithmetic is commonly designated, appears not to have a perfect propriety. The terms "Octaval," "Nonal," "Decimal," "Duodecimal," &c., are derived from the Roman "*ordinals*," and belong to the series Primal, Secundal, Tertial, Quartal, &c. The idea really involved, is not that of relation to a *tenth*, but of relation to a grouping *by tens*, and would require the term "denal" or "denary" — from the Roman "distributive" numerals, of which the terms "binary" "ternary," &c., commence the series.

the language of scientific investigation, and so universally conceded to be one of the most important aims and results of every project of metrical, or numerical improvement.

Upon this basis must the question of so radical a revolution rest. But if it is shown that uniformity in many other relations than those of simple number, and no less vital to the interests and welfare of the race than this boasted uniformity of figures, has constantly and irretrievably been sacrificed to this great idol, — if it is established by the voice of all experience that neither national nor international standards of length, of weight, of area, of volume, or of value, of any single subject, in short, to which these figures can be usefully applied, have ever the slightest hope of obtaining a general authority under the dynasty of this "universal" power, — then must it be dethroned, for very uniformity's sake, and a new dispensation introduced, developed from such principles, and invested with such attributes, that it may rationally be expected to gain at length a universal ascendency, through the concurrent approval and adherence of all intelligent nations. For the attainment of a real uniformity, there seems no other process or alternative; and for such an attainment, no sacrifice of temporary convenience could be held to be too great. The faults of the denary system are too radical to be amended, — too obnoxious to be endured. Sheltered by the inertia and conservatism of inveterate habit, it has been tolerated already much too long. The unskilful contrivance of an early age, it is all unsuited to the wants or uses of an adult manhood of the race.

The *Duodenary* scale has over the denary the advantage of allowing two bisections, and, at the same time, like the senary scale, of admitting of a trisection. Its variety of factors, 2, 3, 4, and 6, give it a much greater power of expressing fractional values than any other scale below it, or immediately above it; and it has accordingly been always found a convenient and favorite number for metrical divisions. The acres, the feet, and the pounds of the Romans were all divided by 12; as are the foot, and the Troy pound, still with us. The signs of the Zodiac, the months of the year, and the hours of the day, have illustrated the number

from the remotest antiquity. In the old French measures of length, the foot was not only divided into 12 inches, but the inch into 12 lines, and the line into 12 points. The "dozen," the "gross" (or 12^2) and even the "great gross" (or 12^3) are widely used in trade at the present day, for the packages of a variety of articles. From the many acknowledged advantages of the duodenary scale, it has found frequent and warm advocates for its adoption as a system of numeration.* In the necessity of two additional integers, it would offer however a considerable increase of complexity and mental labor; while the economy of places in notation could scarcely be regarded as appreciable, — 25 of the duodenary being required for 27 of the denary. As compared with the octonary, it would require 5 places, where the latter would require 6; so that while its digits are more by 50 per cent., the excess of the other in places is only 20 per cent. But there are

* A writer in the *Edinburgh Review*, in expressing a very natural regret that the philosophers of France, when engaged in making so radical a change in the measures and standards of the nation, did not attempt a reform in the popular *arithmetic* as well, appears to regard the duodecimal radix as superior to all others. "The French Academicians"— he says — "though freed at the moment we now speak of, like the rest of their countrymen, from the dominion of the inertia which reigns so powerfully both in the natural and moral world, and gives the time that is past such influence over that which is to come; though delivered from the action of this force in a degree that was perhaps never before exemplified, may be accused, at least in one instance, of having innovated too little, and of having been too cautious about departing from an established practice, though reason was by no means on its side. What we allude to is the system of arithmetical computation, in which they resolved to adhere to the decimal scale, instead of adopting the duodecimal, which from the nature of number, is so evidently preferable. . . . . The property of the number twelve which recommends it so strongly for the purpose we are now considering, is its divisibility into so many more aliquot parts than ten, or any other number that is not much greater than itself. Twelve is divisible by 2, by 3, by 4, and by 6; and this circumstance fits it so well for the purposes of arithmetical computation, that it has been resorted to in all times, as the most convenient number into which any unit either of weight or of measure could be divided. The divisions of the *As*, the *Libra*, the *Jugerum*, the *Foot*, are all proofs of what is here asserted; and this advantage, which was perceived in rude and early times, would have been found of great value in the most improved state of mathematical science. . . . . . We regret therefore that the experiment of this new arithmetic was not attempted. Another opportunity of trying it, is not likely to occur soon. In the ordinary course of human affairs, such improvements are not thought of; and the moment may never again present itself, when the wisdom or delirium of a nation shall come up to the level of this species of reform." (*Edinburgh Review*, Jan. 1807. Vol. 9, page 376.)

far more important considerations than these, which unfortunately oppose themselves to the adoption of this system, as the best substitute for the denary, notwithstanding its admitted features of superiority.

The most fatal objection to the radix 12, is that it permits only a single bisection beyond that given by the radix 10. The quality of continued divisibility, we regard as paramount to all others; not merely for the convenience of art and trade, universal as their requirements are, but even for many scientific purposes; and however valuable the property of a *varied* subdivision, (as that furnished by the duodenary scale,) experience has fully demonstrated, what is clearly seen by theory, that no aliquot parts can ever be as widely useful as the binal fractions. Another objection to the 12 scale, somewhat allied to this, is that the number is not a *power* of any integer,— a point, as we shall discover, of no slight importance. In this respect, it may be remarked, the number nine (awkward and inconvenient as it undoubtedly would be as the basis of an arithmetic,) would have several advantages over the number ten, and even over the number twelve. A third objection to the scale under consideration, which though not so striking, is yet (as we think can be made apparent) no less real, is that the radix is too large. On the simple score of size, there must be somewhere in the indefinite range of scales, a point where we should expect to find the most convenient medium between the inexpediences of opposing difficulties; and although this most advisable limit of magnitude may not admit of very precise determination, the question is one of too great consequence in the comparisons we are making, not to deserve a special attention.

The *Seni-denary* scale presents many excellent points, the number 16 being both a square, and a fourth power, and admitting of indefinite division by two. Its only disadvantage is the incommodious number of digits it would require; while its notation would yet economize only a single place of figures in every six places required by the denary scale.

The *Vicenary* scale furnishes no single point of merit which could recommend it to our acceptance, unless its divisibility by four should

be regarded as giving it a superiority to the denary. With an exceedingly troublesome and unwieldly range of digits, it would reduce the extent of our common notation only from 13 to 10 places. Man was however unfortunately born with 20 extremities, or branches to his limbs, and hence traces of what may be designated a rudimentary vicenary scale, are to be met with among many nations, both ancient and modern. In ancient Phœnicia and Palmyra, the system of numbering by twenties as far as the hundred, prevailed; and from these nations it was derived by the Celtic languages, in all of which, its remains are still found. Among the Scandinavians, also, is found a vicenary numeration. The Greenlanders having counted fingers and toes in periods of five, designated the number 20 by the word *innuk*, which signifies a " man." If they have occasion to proceed higher, the expression for 40 is *innuk arlak*,— " two men,"— &c. A similar method existed among the Aztecs of ancient Mexico; as well as among the tribes of South America. The Teutonic races retain in their languages the traces of the ancient " score," and in parts of England, counting by scores, or twenties, is still quite usual. The translators of our Bible have frequently (though by no means uniformly) introduced this mode of enumeration. Thus we have " threescore and ten," (Ps. xc., 10,) — " threescore and twelve," (Numb. xxxi., 38,) — " threescore and fifteen," (Acts vii., 14,) — " threescore and seventeen," (Judges viii., 14,) &c., &c. The mode of numbering—still in common use in France, also exhibits a very remarkable retention of the antiquated vicenary system.*

This scale is not, as might be supposed, an extension of, and attempted improvement upon the decimal system. On the contrary, it almost universally preceded it; and its employment belongs to the very earliest and rudest stage of barbarian society. It betrays a period of human intelligence, so destitute of all re-

* "The French nomenclature is for the most part purely decimal. The decimal system is observed from twenty (*vingt*) to sixty (*soixante*); here we find a vestige of an old vicenary scale. Seventy, instead of being *septante*, as the decimal system would require, is *soixante-dix*, (sixty-ten); seventy-one, *soixante-onze*, (sixty-eleven); seventy-two, *soixante-douze*, (sixty-twelve,) &c. Eighty, instead of being *octante*, is *quatre vingt*, or four twenties, and ninety is *quatre-vingt-dix*, (four twenties ten); ninety-one, *quarte-vingt-onze*, (four twenties eleven), &c. Thus twenty becomes the radix of the system from sixty to a hundred." (*Lardner's Arithmetic*, page 11.)

source, that fingers and toes must all be pressed into service, to meet the common wants of number; and when these have been exhausted, there has been found among some tribes, no power of thought or word or symbol for aught beyond. It indicates a period long before a conception of any expedient for numerical progressions had dawned upon the savage brain; and hence there is no example of the vicenary scale having ever been extended even as far as to the second place of figures, or to 20 times 20; — nor probably even beyond one hundred. It is evident that when the necessity for expressing larger numbers began to be felt, the cumbrous scale of added "toes," must soon be dropped, and the range restricted to the more manageable mechanism of the ten "fingers." And accordingly, we find the imperfect vicenary to be always overlaid by the denary, with glimpses of the former still appearing through its supplanter.

The *Sexagenary* scale deserves notice only from its historical interest in having been from a very remote antiquity employed for particular purposes among the people from whom we derive our arithmetical notation, — an employment which has been perpetuated throughout Europe and America, to the present, in the smaller divisions of time and of the circle. This scale is of course by far too cumbrous in the range of its units to have ever had a true notation, or to be ever possible as an actual system, founded on its own radix. With its enormous complication of figures, it would still require about $\frac{9}{16}$ (or more than half) of the places of the common system to express its values. It has been found very useful, however, in its limited application, both from the rapidity of its progressions, and from the remarkably varied range of divisibility it permits. The number 60 is divisible by 2, by 3, by 4, by 5, by 6, by 10, by 12, by 15, by 20, and by 30; and has indeed the greatest number of aliquot parts of any number below 120.

Our "minutes," and "seconds" of the degree and of the hour, have thus an Oriental origin. In India, however, from whence the scale was derived, these divisions, as applied to time, had not the same value as with us; as there the day itself was divided into 60 parts, called *guries*, (hours of 24 minutes,) each *gurie* into 60 parts,

called *polls*, (minutes of 24 of our seconds,) and lastly each *poll* into 60 *mimiks*, or twinklings of an eye, (four-tenths of a second.) It is believed that this division of time is retained by the Hindoos to the present day. They also employ a period of 60 years, as we do the century.

In its astronomical application this scale has been found exceedingly useful. The properties of the circle require that it should be frequently divided into sixths, as well as into quarters; the sixth being, as is well known, the radial arc,—or that whose chord is exactly equal to its radius. The zodiacal, or ecliptic circle of the heavens had from the earliest antiquity been divided into twelfths, a period representing approximately the movement of the sun during one lunation. As this comprised very nearly 30 days, the "sign" became naturally divided into 30 degrees; and this expresses so closely the arc of the earth's orbit described in one mean solar day, that when the earth is moving slowest, (or at its aphelion,) it falls but 3 minutes within one degree, and when it is moving fastest, (or at its perihelion,) it exceeds the degree by only a single minute. The radial arc of two "signs," or 60 degrees, suggested its own subdivisions. Hence was derived the table of 60 seconds to the minute, 60 minutes to the degree, and 60 degrees to the sextant,—6 of these comprising the circle. This system, answering so well the requirements of various division, was introduced from India into the Alexandrian school, by the illustrious Ptolemy,* who did so much toward giving astronomy a scientific form. The sexagenary scale has never, however, been computed by any other than a denary radix. It must excite surprise, therefore, that the Hindoo notation of the scale was not also introduced by Ptolemy at the same time; and the world thus put in possession of this grand invention eight centuries earlier than it was by the Arabic importation.†

* Although the sexagesimal arithmetic is commonly ascribed to Ptolemy, it is probably an Eastern invention. The Indians to this day, employ the sexagesimal division of time." (*Edinburgh Encyclopedia*, art. "Arithmetic.")

† The Greeks, like the Hebrews, Arabs, and all other nations excepting the Hindoos, employed an alphabetic numeral; and it is a somewhat curious circumstance that our modern character for the cipher, was derived not from India or Arabia, but from Alexandria. The Hindoos indicated the cipher place by a simple dot, (·) and the Arabians, in borrowing their system, did the same; until the sexagenary system, introduced by

In our survey of the principal scales, from which alone a selection could be made for popular uses, we have found that there are certain incidental, but opposing advantages, incompatible with each other; and that no scale, therefore, could possibly furnish a maximum of every condition that might be thought desirable. Thus the binary scale affords so admirable a simplicity, beauty and facility, that it would have to be regarded the perfect system, did not its redundant employment of figures, (the necessary consequence of its simplicity,) render it unsuited to the small and constant calculations required in the daily course of trade. On the other hand, the manifold divisions permitted by the sexagenary scale, give it convenient qualities, impossible to the lower scales; but here we find a complication so onerous that it would appal the most inveterate of calculating monomaniacs.

The conditions, however, that are really most essential to an arithmetical radix, are so few and precise, and their requirements so imperative, that there is little difficulty in deciding upon "the best possible scale of numeration." The first consideration would naturally have regard to the size of the radix, in order to assign certain limits within which our scale is to be found. To realize a maximum convenience, it must be neither too large, nor too small. We have seen that while the notation of places (and the consequent labor of transcription,) diminishes very slowly with the ascending scales, — the tax upon the mental faculties, increases in a far more rapid ratio. The labor of mere *calculation*, which may be estimated at zero for the binary scale, advances materially, and in a compound ratio with every figure added to the radix. Were we then required to choose between any two scales — separated by

Plotemy so many centuries before, supplied them with a new character. This philosopher, finding a frequent occasion to mark the absence of a particular denomination, (as "no minutes," or "no seconds,") in order to avoid mistake, employed the first vacant letter of the alphabet for that purpose. As the Greek numeral for 60 is the letter ξ all those which followed would be useless for the sexagenary scale; hence the next letter, ο (omicron) naturally became the empty counter. This notation became established by long habit among the astronomers of Alexandria, Constantinople, and Arabia ; and finally crept into the Hindoo system of numerals. Thus to the accidental position of the Greek letter *omicron*, which happened to represent *seventy*, we are indebted for the present form of our modern cipher, as a circle, instead of a decimal period.

a considerable interval, that is, between a very small one and a very large one, (no other insuperable objection being supposed) we should adopt, unhesitatingly, the smaller one. The advantage imagined by some, of the great expressiveness of a rapid increase of value, is wholly illusory. It needs comparatively very few figures, in any case, to carry us not only beyond all true conceptions of magnitude, but beyond all rational requirement of any real calculations we can devise.* There is, in the law of continued geometrical progression, even on its lowest scale, a power so overwhelming, that we feel we have no extra wonder or admiration left to spare, upon these "infinites of higher order," and confess to a predilection not to travel at such dizzying speed.

The world has had some centuries of experience in the denary arithmetic. We are all familiar with the laborious and tedious discipline by which its practice is acquired; and we are all conscious of the exertion of thought demanded to perform a lengthy operation in figures. When we consider the amount of time bestowed in training youth in this branch of learning; and yet the fact that not one half so trained, are really expert in calculation, we must record it as our deliberate conviction, that *the denal radix is too large*. We believe that a lower figure would give the true desideratum — the *minimum* of labor. Nay, as between the scale of

* To give an illustration of the truly infinite power of figures, let us strain our imagination to devise some overwhelming product. Light, which (were it prone to devious courses,) would flash around our globe in eight completed revolutions during one single tick of the clock's pendulum, would travel in one hundred thousand years, only 607 thousand billions of miles, (a period of 18 figures) — or multiplied by the inches in the mile, — 38,463 trillions of inches; (requiring 23 figures.) According to the received data of the undulatory theory, the extreme violet ray of the luminous beam (the most rapid of the prismatic spectrum,) vibrates 727 millions of millions of times in a second! Let us suppose the remotest of the telescopic nebulæ to be in light measure, one hundred millions of years distant; — how many undulations must its light perform before reaching our eye? Nay, as we can only conceive of the material or stellar universe as a finite extension (or point) in the infinite void, let us suppose (for definiteness of idea!) that the confines of this entire *Cosmos* were sixty thousand times more distant than this faintest nebula,— or six billions of years, — even *then* the undulations of light throughout that unimaginable space, would be expressed by 137,651 quintillions; (or 36 figures.) Or if we wing our flight as far in the opposite direction, we do but multiply these figures by 2, for the whole extent, — without even adding a single place of figures to express this entire diameter! (275,302 quintillions). And this may all be written down with ease — upon the thumb-nail!

ten and that of *six*, we incline to the opinion that the latter would be found the more convenient notation. Its labor, both of acquisition and of exercise, would certainly be far less than half; while its figures in use would only be about a third more. *A priori*, we might expect that a scale established in rude and inexperienced times, (were it not that it was really determined by an arbitrary and extraneous circumstance) — would be too large in its ratio of progression, — rather than too small; and that a more enlightened age would find it convenient to reduce it; just as we have seen to occur with the vicenary and the denary scales, in their early history.

The second essential that should be demanded in a radix is that it must admit of indefinite bisection, or in other words, that it must be found among the powers of two; namely, 4, 8, or 16. As 4 is probably too small, and 16 certainly too large, — we have the octonary scale alone left to satisfy our two most vital conditions of a medium size, and a complete divisibility. The concurrence of these qualities in any one scale, and in that one alone, is sufficient to establish its claims against all competitors. There is but one scale which could have any pretensions to be considered a rival, or which would be likely to find intelligent advocates; and that is the duodenary. Much stress has been laid upon the number of its aliquot parts. That this quality is a highly useful one, we frankly acknowledge, but yet, as we maintain, not near so useful as that other quality this radix lacks, the facility of successive halving. The number 12 is not a power; the number 8 is a cube; — an important advantage in several respects, but particularly in the application of this scale to a system of metrology, from the simple relations thereby established between the measures of length and those of volume, — by which both weights and measures of capacity are determined. All that has been said on the subject of the denary being too large a scale, applies with much greater force against the duodenary. And, finally, we believe that a large majority of the mathematicians would give their vote unhesitatingly in favor of the octonary arithmetic. It appears to combine advantages of the very first importance, and those impossible in any other scale.

While perfectly adapted to the highest requirements of science, it is as exactly suited to the trivial wants and petty occasions of our daily life. It possesses a degree of simplicity the most attainable without a sensible increase of figuring. The simple suppression of the two largest digits of our common systen (8 and 9) throughout every place of figures, would be found, we think, to reduce the working labor by at least one half. In choosing between a radix of a second power, (as 4,) and one of a third power, the latter would for several reasons be preferred. It would undoubtedly be advantageous for it to be at the same time both a square and a cube. But unfortunately we can meet with no such favored number, until we reach the period 64. Our octonary radix is, therefore, beyond all comparison the "*best possible one*" for an arithmetical system.

After this somewhat tedious preparatory exposition, we now propose to briefly develop the scale of numeration thus selected; and to derive from it an ideal system of measures, based throughout upon the leading ideas of the French system;—availing ourselves, as we believe, of every beauty and refinement offered by it, and avoiding, as we are no less firmly persuaded, every difficulty, and defect inherent in it. Let us attempt to employ our proposed scale of number in the first place, by putting it in an intelligible form. Although we might readily discriminate between the octonary and the denary notation by the simple expedient of using a somewhat different type, of our common figures, (suppressing the 8 and the 9,) yet even with this device, the association of local value is so strong that it would not be easy to avoid confusion of idea in attempting to read and understand the unfamiliar conversion. It will be found much easier, therefore, to devise a set of characters for the octonary scale; which should be entirely distinct both from the letters of the alphabet, and from our ordinary figures. To assist us still more in reading them, these characters might be made significant symbols, by the number of lines employed in the construction of each, though this would be a matter of very little importance in a form of character that should be permanently adopted. The characters should all be simple; they should all have the same size, for the obvious convenience of the typographic "dress;" and they

should be so distinctive, that no one could easily be mistaken for another. Let us then represent *one* by L; *two* by C; *three* by 6; *four* by F; *five* by P; *six* by G; and *seven* by B; the cipher having no intrinsic value, may very well continue to be still represented by 0. Our eight *digits*, then, (if we must still use so barbarian and unmathematical a designation,)* would stand thus: 0LC6FPGB.

In reading these octonary numbers, a distinctive name for each, as well as for the places occupied by them, would become even more necessary than a distinctive form. The terms "ten," "hundred," "thousand," especially, are too essentially decimal in their origin, and too ineffaceably stamped by usage in their significance, to permit their use in any novel application. The names, like the symbols, should be both as simple and as distinct as possible. The simplest name is a monosyllable, containing but one consonant, and one vowel sound. Let this then be the rule of our numerical vocabulary. It will be convenient and even advisable to preserve a resemblance to the popular numerical language, that the analogy of structure may be the more apparent. The word *one*, will naturally give us the French "un,"—*two*, will give us "du;"—*three* will give us "the;" the consonant sound being really a simple one, although requiring two letters in our language. The word "tre" would have been better, as being very near the Latin *tres*, the Greek *treis*, and the original Sanscrit *tri*; † but the double conso-

* It has been sometimes remarked by advocates of the octonary arithmetic, that if our stupid ancestors had only used their thumbs as the counters of the digits, they would have found that they had but eight fingers, and we should then have had the octaval period—"founded in nature." It may be supposed from the preceding discussion of this subject, that we attach but little importance to such a consideration.

† It is a matter of curious philological interest to trace the Sanscrit or ancient Indian parentage of all our modern European languages; especially in the names of the numerals. In this particular the different vocabularies of the numerous and wide-spread races,—of the Celtic, the Romaic, the Sclavonic, and the Gothic, with its two great families of the Scandinavian, and the Teutonic,—appear only as dialects of each other. The names of the first ten numbers, in a few languages, are here selected, mainly from the Introduction to Bosworth's Anglo-Saxon Dictionary.

nant excludes it under our rule. The word *four* will give us "fo;" but for five, in order to avoid a consonant recurrence, we shall have to resort to the original Sanscrit, *pancha*, which will give us "pa." Our *six* will give us "si" or "se;" but for our next number, as we can derive no satisfactory help from English or Latin, Greek or Sanscrit, we are driven to some arbitrary syllable. As seven is the last of our series, we may accept the single independent term with less reluctance; and that its sound may be as distinctively marked as possible, let us call it "ki."

Here, then, we have assigned for each of the numerals "a local habitation and a name."

L *Un;* C *Du;* 6 *The;* F *Fo;* P *Pa;* 6 *Se;* 8 *Ki.*

Our decades — twenty, thirty, forty,— offer us the very suitable and simple suffix "ty" to designate our octades. Our hundred suggests the syllable "der" as a convenient designation of the third place

Sanscrit.	Persian.	Greek.	Roman.	Welsh.	Gothic.	German.	Saxon.	English.
Aika	*yika*	*eis,en*	*unus*	*un*	*ains*	*ein*	*an*	*one*
Dwau . . .	*du*	*duo*	*duo*	*dau*	*tvai*	*zwei*	*twa*	*two*
Tri	*seh*	*treis*	*tres*	*tri*	*threis*	*drei*	*threo*	*three*
Chatur . . .	*chehaur*	*tessares*	*quatuor*	*pedwar*	*fidvor*	*vier*	*feower*	*four*
Pancha . .	*pendj*	*pente*	*quinque*	*pump*	*finf*	*funf*	*fif*	*five*
Shash. . . .	*shesh*	*hex*	*sex*	*chwech*	*saihs*	*sechs*	*six*	*six*
Saptan . . .	*heft*	*hepta*	*septem*	*saith*	*sibun*	*sieben*	*seofen*	*seven*
Ashta. . . .	*hesht*	*okto*	*octo*	*wyth*	*ahtan*	*acht*	*eahta*	*eight*
Navan . . .	*nuh*	*ennea*	*novem*	*nan*	*niun*	*neun*	*negon*	*nine*
Dashan . .	*deh*	*deka*	*decem*	*deg*	*taihun*	*zehen*	*tyn*	*ten*

That these Sanscrit terms should have been so widely diffused, while yet no traces of the Hindoo arithmetical notation should ever have been found outside of India, would seem to show that this derivation was antecedent to the formation of a written language; or, at least, prior to the invention of the *cipher*. A nomenclature may be easily transmitted orally or by tradition: a notation could be communicated and preserved only by records.

We may here remark, that to the Sanscrit we are indebted for the denominations of our two lowest coins. From the Sanscrit *Sata* or *Shatum* (a hundred), through the Latin *centum* we obtain our "cent;" and from the Sanscrit *Dasa* or *Dashan* (ten), through the Latin *decem*, and the French *disme* or *dime*, we obtain the name of our "ten-cent piece."

of figures; and our thousand, will give us "sen." And here we may improve upon our present mode of expressing "places" by employing these distinctive suffixes, as independent nouns, significant of a particular order of units, without reference to any special or intrinsic value. Thus a simple unit would indicate any figure occupying the first place; a Ty would indicate any figure occupying the second place; a Der, any figure occupying the third place, &c.

But mindful of that prudent law — "economy of means," — and not to burden our infant scheme with too great a load of unfamiliar nomenclature, (always the greatest obstacle to the reception of any novel system,) let us resort to combinations of these simple suffixes, instead of applying a new term to each new place of figures. By this means we shall be required to introduce new terms, only at the successive and advancing powers of each great unit. Thus using "Ty" for the second place, and "Der" for the third place, we may very well employ the word "Ty-der" for the fourth place, "Sen" for the fifth place, "Ty-sen" for the sixth place, "Der-sen" for the seventh place, and "Ty-der-sen" for the eighth place. Here is a pause; and to do honor to the number *eight*, — this should comprise one independent period of figures; to be followed by a new term, the analogue of our MILLION.* We cannot derive a convenient suffix, however, from this term; we shall therefore have to coin a new one. Let us call our great figure KALY. We have thus the progression; one "Ty" squared, is one "Der;" one "Der" squared, is one "Sen;" one "Sen" squared, is one "Kaly;" the intermediate places being expressed by the obvious compounds of these words. Or to illustrate the series proposed, by our own decimal terms, it is as though having assigned eight places of figures instead of six, for our million origin, we should reach it by this scale: tens, hundreds, ten-hundreds, thousands, ten-thousands, hundred-thousands, ten-hundred-thousands, millions:

* Our *Million*, the square of the Roman *Mille*, is a comparatively modern word; and useful as it is now universally esteemed, it appears on its first introduction to have met with but little favor. "Bishop Tonstall, who has discussed at great length the Latin nomenclature of numbers, speaks of the term *million* as in common use, but he rejects it as barbarous." (*Peacock's Arithmetic.*)

the "ten-hundreds," and the "ten-hundred-thousands," being interpolated places.

Words manufactured to meet a new want, have always a somewhat barbarous and uncouth sound, until familiarized by custom; and are usually received but slowly and with reluctance. Unless they can boast a pedigree and a history, they must expect from the world, like other *parvenus*, no very cordial greeting. From the habits of thought of a very large majority of mankind, it is found so much easier to use old words in a double sense, than to accept the precision of a new phraseology, that there is little doubt the octonary notation could be much more readily taught (except to children) by simply erasing the figures 8 and 9, from the common arithmetic. That it is more philosophical, however, to assign to everything its own appropriate name, can scarcely need a formal statement; and if the system now proposed have the high claims and merits we have represented, we think that no apology is required for the attempt to clothe it in a fitting garb. We here present accordingly the numeration table, as resulting from the names we have just above suggested :

Numeration Table.

L, Un = 1	LL, Unty-un = 9	CL, Duty-un = 17	ΘL, Thety-un = 25
C, Du = 2	LC, Unty-du = 10	CC, Duty-du = 18	ΘC, Thety-du = 26
Θ, The = 3	LΘ, Unty-the = 11	CΘ, Duty-the = 19	ΘΘ, Thety-the = 27
P, Fo = 4	LP, Unty-fo = 12	CP, Duty-fo = 20	ΘP, Thety-fo = 28
P, Pa = 5	LP, Unty-pa = 13	CP, Duty-pa = 21	ΘP, Thety-pa = 29
Б, Se = 6	LБ, Unty-se = 14	CБ, Duty-se = 22	ΘБ, Thety-se = 30
B, Ki = 7	LB, Unty-ki = 15	CB, Duty-ki = 23	ΘB, Thety-ki = 31
LO, Unty = 8	CO, Duty = 16	ΘO, Thety = 24	PO, Foty = 32
PL, Foty-un = 33	PL, Paty-un = 41	БL, Sety-un = 49	BL, Kity-un = 57
PC, Foty-du = 34	PC, Paty-du = 42	БC, Sety-du = 50	BC, Kity-du = 58
PΘ, Foty-the = 35	PΘ, Paty-the = 43	БΘ, Sety-the = 51	BΘ, Kity-the = 59
PP, Foty-fo = 36	PP, Paty-fo = 44	БP, Sety-fo = 52	BP, Kity-fo = 60
PP, Foty-pa = 37	PP, Paty-pa = 45	БP, Sety-pa = 53	BP, Kity-pa = 61
PБ, Foty-se = 38	PБ, Paty-se = 46	ББ, Sety-se = 54	BБ, Kity-se = 62
PB, Foty-ki = 39	PB, Paty-ki = 47	БB, Sety-ki = 55	BB, Kity-ki = 63
PO, Paty = 40	БO, Sety = 48	BO, Kity = 56	LOO, Under ≐ 64

It will be seen by this table that we have no peculiar word corresponding to the " ten " of the denary scale ; and this we regard as an advantage, not only in being more systematic, but in giving greater precision of expression and idea. Instead of using the same word to indicate both a place, or local value (as the " ten-place,") and a specific number, we are furnished with two distinct words, — " Ty " designating the place, and " Unty " specifying one in the ty-place, as " Duty " specifies two in the ty-place. All that is needed to carry out this system is to add a table of places.

Notation Table.

Units.	Ties.	Ders.	Tyders.	Sens.	Tysens.
L Un	L0 Unty	L00 Under	L000 Untyder	L,0000 Unsen	L0,0000 Untysen
C Du	C0 Duty	C00 Duder	C000 Dutyder	C,0000 Dusen	C0,0000 Dutysen
θ The	θ0 Thety	θ00 Theder	θ000 Thetyder	θ,0000 Thesen	θ00000 Thetysen
F Fo	F0 Foty	F00 Foder	F000 Fotyder	F,0000 Fosen	F0,0000 Fotysen
P Pa	P0 Paty	P00 Pader	P000 Patyder	P,0000 Pasen	P0,0000 Patysen
6 Se	60 Sety	600 Seder	6000 Setyder	6,0000 Sesen	60,0000 Setysen
8 Ki	80 Kity	800 Kider	8000 Kityder	8,0000 Kisen	80,0000 Kitysen

Dersens.	Tydersens.	KALIES.
L00,0000 Undersen	L000,0000 Untydersen	L,0000,0000 Unkaly
C00,0000 Dudersen	C000,0000 Dutydersen	C,0000,0000 Dukaly
θ00,0000 Thedersen	θ000,0000 Thetydersen	θ,0000,0000 Thekaly
F00,0000 Fodersen	F000,0000 Fotydersen	F,0000,0000 Fokaly
P00,0000 Padersen	P000,0000 Patydersen	P,0000,0000 Pakaly
600,0000 Sedersen	6000,0000 Setydersen	6,0000,0000 Sekaly
800,0000 Kidersen	8000,0000 Kitydersen	8,0000,0000 Kikaly

The Unkaly is the eighth (or untieth) power of Unty. Its value is 16,777216 ; and it requires but one more figure to express this large amount, than is required by the denary scale. A second place of figures is not lost by our new system, — that is, its notation does not exceed that of the common system by two places, until the number

8589,934592 is reached; these 10 figures requiring 12 (namely L and eleven ciphers) in the octonary scale to represent their value. If this should appear surprising to any, it must be remembered, that although at 8, and at 64, an additional figure is required by the octonary system, yet after 10 and 100, the denary also requires this additional figure; and considering this, we shall find that the two scales are *equal* in the number of places occupied, — from 1 to 7, inclusive, — from 10 to 63, inclusive, — from 100 to 511, — from 1000 to 4095, — from 10,000 to 32767, — from 100,000 to 262143, — from 1,000000 to 2,097151, — from 10,000000 to 16,777215, — from 100,000000 to 134,217727, — and for the last overtaking, from 1000,000000 to 1073.741823. After this long continued chase, the octonary scale at the next figure, or 1073,741824, (Under-Kaly) loses a place which is never regained. It may not be uninteresting to add, that this scale does not obtain an excess of *three* places until it reaches the enormous number of 9 trillions, 223372 billions, 036854 millions, 775808, these 19 figures being expressed by L and 21 ciphers. This amount diminished by a single unit, or by the last figure 8 being exchanged for a 7, is expressed in the octonary system by 21 *kis* (B) which would be an excess of only *two* places of figures.

Turning from this comparison of the relative powers of the two scales, to their relative simplicity, as exemplified by the octonary multiplication table, we shall find the contrast here as striking as was their parity on the other hand remarkable.

Multiplication Table.

	C	θ	F	P	6	B
B	L6	CP	θF	Fθ	PC	6L
6	LF	CC	θ	θ6	FF	
P	LC	LB	CF	θL		
F	LO	LF	CO			
θ	6	LL				
C	F					

The mere inspection of this table,* is sufficient to show, that the time and labor of acquiring it would not be half that required for committing to memory our received form; and this facility of acquisition would include almost a corresponding degree of readiness in its use. Figures, like furniture stored in the chambers of the brain, require a constant attention and arranging, to be kept in state for use; and the amount of care and trouble unconsciously bestowed upon them, must be proportioned to the number of the pieces after which we have to look. We think, therefore, that it is no idle boast to say that a child could be taught a thorough knowledge of the four great rules of arithmetic, and a ready skill in their practical applications, through the octonary system, in one half the time required for obtaining an equal knowledge and skill by the common system. Nor is this simplification of arithmetical operations its only merit. The danger of error increases rapidly with the increasing complexity of the numeric scale; and there is no doubt that our new system would ensure an increase of accuracy, at least equal to its ratio of simplicity. And if to this were added the facility which would result from constructing all our tables of weight and measure upon this scale — (a scale so admirably suited to them,) and thereby entirely discarding the whole tedious and troublesome practice of "reduction," from our Arithmetic, — the economy of time and labor would be something quite astounding.†

* This table might be expressed in our own figures, in the following form, remembering that these figures have all an octonary value:

$2 \times 2 = 4$					
$2 \times 3 = 6$	$3 \times 3 = 11$				
$2 \times 4 = 10$	$3 \times 4 = 14$	$4 \times 4 = 20$			
$2 \times 5 = 12$	$3 \times 5 = 17$	$4 \times 5 = 24$	$5 \times 5 = 31$		
$2 \times 6 = 14$	$3 \times 6 = 22$	$4 \times 6 = 30$	$5 \times 6 = 36$	$6 \times 6 = 44$	
$2 \times 7 = 16$	$3 \times 7 = 25$	$4 \times 7 = 34$	$5 \times 7 = 43$	$6 \times 7 = 52$	$7 \times 7 = 61$

† "It is impossible to estimate with any degree of accuracy," says Mr. Nichol, "the amount of labor annually thrown away by the nation at large, while persisting in performing the manifold computations necessary to its gigantic commerce and industry, by means of a series of tables so needlessly complicated and imperfect as those now in

Our exposition of the subject of numeration has been so extended that neither time nor space will now permit us to illustrate the practical working of the arithmetical system here proposed, as we had intended. It is evident, however, that we are here equipped with a mechanism fully adequate to the resolution and expression of all arithmetical operations. Framed by a strict analogy with our present system, it affords us every facility and advantage that this can boast; and differing from it only in the number of its integers, it relieves us entirely from the difficulties and embarrassments which have ever been the opprobrium of our decimal scale. Merely to exhibit the form and method of our scheme, we may here indicate that the present year "1859" would in the octonary style be expressed 6P06 : — *Thety pader and the.* The diameter of the earth (7,925 miles,) would be expressed L,8E6P : *Unsen, Kity theder setypa ;* or in feet (41,847088) — C,6BP0,F6CF : — *Dukaly, thety kider patysen, foty seder duty-fo.*

We now proceed as rapidly as possible to the application of this improved numeration to the determination and distribution of a system of weights and measures. Of all the systems of metrology yet perfected, or even proposed, that of the French, is in the philosophical character of its standards, as well as in the ingenuity, simplicity, and precision of its details, undoubtedly by far the most admirable, and the most worthy of our imitation. "The French system," says Mr. Adams in the excellent Report on Weights and Measures, from which we have already more than once had occasion to quote, "embraces all the great and important principles of uniformity which can be applied to weights and measures. *But that system is not yet complete ; it is susceptible of many modifications and improvements.* Considered merely as a labor-saving machine, it is a new power offered to man incomparably greater than that which he has acquired by the new agency

use. But the waste of time and loss of money must be something quite enormous, while every day it becomes greater and greater. Were the different denominations of weights, measures and money brought into harmony with the fundamental principle of our common arithmetic, it may be safely affirmed that the labor of commercial and professional calculations would be reduced much below one-half of what is now expended in this direction, while the risk of errors would be diminished in a still greater ratio." (*Encyclopedia of the Physical Sciences*, art. Weights and Measures, page 778.)

which he has given to steam. It is in design the greatest invention of human ingenuity since that of printing. But like that and every other useful and complicated invention, it could not be struck out perfect at a heat. Time and experience have already dictated many improvements of its mechanism. But all the radical principles of uniformity are in the machine. . . . *Uniformity* of weights and measures, — permanent, universal uniformity, adapted to the nature of things, to the physical organization and to the moral improvement of man, — would be a blessing of such transcendent magnitude, that if there existed upon earth a combination of power and will, adequate to accomplish the result by the energy of a single act, the being who should exercise it would be among the greatest of benefactors of the human race. The glory of the first attempt belongs to France. France first surveyed the subject of weights and measures in all its extent and all its compass. France first beheld it as involving the interests, the comforts, and the morals, of all nations, and of all after ages. . . . . In freely avowing the hope that the exalted purpose first conceived by France may be improved, perfected, and ultimately adopted by the United States and by all other nations, equal freedom has been indulged in pointing out the errors and imperfections of that system, which have attended its origin, progress, and present condition."

Looking at the French *metre* simply as a practical material standard, (without any reference to its ideal derivation,) the first criticism we would naturally have to make upon it, is that it gives us a measure most unfortunate in its size. Like our own awkward yard stick, it is not such a rule as we can conveniently carry about with us: * it is not such a rule as we would measure our houses

* "Perhaps for half the occasions which arise in the life of every individual, for the use of a linear measure, the instrument to suit his purposes must be portable, and fit to be carried in his pocket. . . . . For all the ordinary purposes of mensuration, excepting itinerary measure, the metre is too long a standard unit of nature. It was a unit most especially inconvenient as a substitute for the foot, a measure to which, with trifling variations of length, all the European nations and their descendants were accustomed. The foot rule has a property very important to all the mechanical professions, which have constant occasion for its use: it is light, and easily portable about the person. The metre very suitable for a staff, or for measuring any portion of the

by, or the furniture within them; it is not such a rule as the draughtsman could employ, or the carpenter cut, or lay off his boards by; it is not such a rule as the mechanic could use in his workshop, or the machinist handle in fitting his engines. In short, as a popular standard, it is utterly useless except on the counter of the draper. Theoretically, it matters little whether our unit of reference be the inch or the mile; but for the practical business of daily life it becomes a matter of the very highest importance, that our unit of measure should be such a one as shall have the most convenient and universal application.

Two standards only have ever had a general use and currency, — the *cubit* and the *foot*. Both derived from the human person, it is natural they should be found the most useful measures for the common wants of the person. The cubit may be said to be almost a natural standard; and it is the most ancient of measures, while it is still prevalent throughout the orient. Universal, or nearly so, throughout the nations of antiquity, — it was the common measure of the Israelites, and is referred to in their earliest records. The ark is measured by cubits (Gen. vi. 15,) and the height of the flood is in cubits. Goliath's height was six cubits and a span. The temple of Solomon is measured in cubits; and walls of cities are measured by the same. (2 Kings xiv. 13.) The foot appears to be a much later standard of measure. Introduced by the Greeks and Romans, it has prevailed in modern times wherever the Roman influence has been felt.*

earth, has not the property of being portable about the person; and for all the professions concerned in ship or house building, and for all who have occasion to use mathematical instruments, it is quite unsuitable. It serves perfectly well as a substitute for the yard or ell, the fathom or perch; but not for the *foot*. This inconvenience, great in itself, is made irreparable when combined with the exclusive principle of decimal divisions. The union of the metre, and of decimal arithmetic, rejected all compromise with the foot. There was no legitimate extension of matter intermediate between the ell and the palm, between forty inches and four. This decimal despotism was found too arbitrary for endurance." — (*Adams's Report on Weights and Measures.*)

* "The cubit was also a primitive measure of length among the Greeks; but at the institution of the Olympic games by Hercules, his foot is said to have been substituted as the unit of measure for the foot-race. Six hundred of these feet constituted the stadium or length of the course or stand, which thenceforth became the standard itinerary measure of the nation. It was afterwards by the Romans combined with the pace, a thousand of which constituted the *mile*. The foot and the mile, or thousand

If the foot has been found a more manageable multiple of both the pace and the fathom or its half — the ell, — than the cubit, we are disposed to regard the latter as the more beautiful and useful rule, and the more convenient unit of length. Certainly the occasions are not unfrequent, when we need the addition of a few inches to our foot-rule to measure common objects. At all events in selecting a standard, adapted to the popular wants, it may be regarded as tolerably manifest, that its length should not be less than a foot, and that it should not exceed two feet, — the common carpenter's rule. The cubit is the mean between these extreme limits.

This consideration brings us to the derivation of the standard. "In all the proceedings" says Mr. Adams, "whether of learned and philosophical institutions, or of legislative bodies relating to weights and measures within the last century, an immutable and invariable standard from nature, of linear measure, has been considered as the great desideratum for the basis of any system of metrology. It is one of the greatest merits of the French system to have furnished such a standard for the benefit of all mankind. . . . . In the establishment of the French system, the pendulum, as well as the meridian, has been measured; but the *standard* was after a long deliberation, after a cool and impartial estimate of the comparative advantages and inconveniences of both, definitively assigned to the arc of the meridian, in departure from an original prepossession in favor of the pendulum." A writer in the Edinburgh Review for Jan., 1807, remarks: "Three different units fell under the consideration of these philosophers, to wit, the length of the pendulum, the quadrant of the meridian, and the quadrant of the equator. If the first of these was to be adopted, the commissioners were of opinion that the pendulum vibrating seconds in the parallel of 45 degrees deserved the preference,

paces, are our standard measures of length at this day. The foot has over the cubit, the advantage of being a common aliquot part of the pace and the fathom; and affords the natural means of reducing the two standard measures of length to one. Its adoption was therefore a great and important advance towards uniformity; and this may account for the universal abandonment by all the modern nations of Europe, of that primitive antediluvian standard of measure, the cubit." — *(Adams's Report on Weights and Measures.)*

because it is the arithmetical mean between the like pendulums in all other latitudes. They observed, however, that the pendulum involves one element which is heterogeneous, to wit, time; and another which is arbitrary, to wit, the division of the day into 86,400 seconds. It seemed to them better that the unit of length should not depend on a quantity, of a kind different from itself, nor on anything that was arbitrarily assumed. The Commissioners therefore were brought to deliberate between the quadrant of the equator, and the quadrant of the meridian; and they were determined to fix on the latter, because it is most accessible, and because it can be ascertained with the most precision." (*Edinburgh Review*, Vol. 9, p 379.) *

That this selection was wise at the time it was made, cannot be doubted. That it would be wiser now to select the equator, can we think be made equally evident. By the modern methods of electro-magnetic determination of longitude, an arc of the equator could now be ascertained with as much accuracy, as one of a meridian, and perhaps with even greater precision. A national, or, what would be far nobler, an international commission, liberally endowed with every needed equipment, for measuring in South America, and in Africa, arcs of the equator,—if possible entirely across either continent; and also (what would be very important) one through the opposite island of Borneo,—is an enterprise due to the enlightened spirit and scientific progress of the age, and would be one worthy of the united wisdom and resources of the three greatest nations of the world. The determination of the precise figure and dimensions of our globe,—that fundamental problem of practical astronomy,—is one of such transcendent importance, that no outlays should be regarded as injudicious or misapplied, that would offer the prospect of even a slight improvement in the accuracy of our results.

The equator is, in the first place, undoubtedly the true girth and measure of the earth; and the circumference should always be understood to be this natural measure, unless otherwise specified. In the next place, the meridian not being a circle (owing to the

* See note C—appended to this Report.

polar flattening of the earth,) no two degrees of its quadrant have exactly the same value ; which renders the estimates of its degrees exceedingly awkward. According to the computations of Mr. T. J. Cram, (Silliman's Journal of Science for 1837, vol. 31, page 230,) one degree of latitude at the pole is equal to 69.39759375 miles, while one degree at the equator is only 68.70859375 miles, — a difference of more than two thirds of a mile ! In addition to all this there is some reason for doubting whether different meridians are uniform in length and curvature. An arc of the meridian south of the equator, measured in 1752, by Lacaille, (at the Cape of Good Hope) gave very unsatisfactory results.*

But through the reductions of various eminent mathematicians we have now the equatorial circumference of the earth as well and accurately determined as any other measure of it. The two best and most recent determinations of the earth's equatorial diameter, are those of Bessel, and Airy, who, by independent calculations, agree in the value 7925.6 miles, and differ only by 234 feet! Bessel making it 41,847,192 feet, and Airy, 41,847,426 feet. The mean of these results will give us 131,467,196 feet, as probably a very close measure of the earth's equator. We have every reason, therefore, for deducing our standard of measure from this line,— the only true circle by which the earth is circumscribed; — we have none for going back to the irregular meridian.

In no particular has the decimal principle of the French system proved so signal and utter a failure, as in its application to the division of the circle. We have already noticed that the sixth part of the circle is one of its most fundamental divisions, — one which cannot be neglected for any theoretical advantage of adherence to

* The result of Lacaille's measurement made a degree at the Cape of Good Hope nearly the same as in France, ten degrees further from the equator. " On account of the anomalous result of this measure, indicating a dissimilarity of figure in the Northern and Southern hemispheres, doubts have frequently been entertained of its accuracy." (*Penny Cyclopedia*, vol. 25, page 215, art. " Trigonometrical Survey.") " After the lapse of nearly a century, the arc has been lately re-measured under the auspices of the British Government; and an approximate calculation of the observations shows that the greater part of Lacaille's anomaly was produced by *mountain attraction* on his plumb line, (1855.)" (*Encyclopedia Britannica*, 8th edition, vol. 9., page 550, article, " Figure of the Earth," by Th. Galloway.)

system. We have seen, moreover, how admirably our present division of the quadrant into 90 parts or degrees, answers all the various purposes required. In adding ten more degrees to the whole, so as to make an even hundred, the French philosophers sacrificed completely its primary and beautiful relations. The sextant no longer had a possible expression in the centesimal scale. A very brief experiment demonstrated, what should have been clearly anticipated without it, that the new degrees were wholly impracticable. This part of the system was therefore speedily and universally abandoned: * and yet this was really a surrender of the very foundation of the metrical division.

The *metre* had been made the 10 millionth part of the quadrant, that the new degree might just represent 10 myriametres: but the abolition of this ideal degree, left the myriametre (and with it of course the metre) a most inconsequential and unmeaning unit. So that now the kilometre no longer represents a minute, and the decametre, a second, as was its original plan and purpose.

The selection of the meridian, we may observe, necessarily involved a reference to its natural fraction, the quadrant; the distance from pole to equator: but had the equator been the standard chosen, inasmuch as it has no such natural measure, the sextant of it might just as properly have been made the starting unit, as its quadrant. And this would have escaped the principal difficulty: for the sextant will easily supply us with a multiple of the quadrant, — though the latter may not conversely, so readily commensurate the former. Instructed by such distinguished failures, let us then start with the sextant of the equator as our prime unit of measure. We shall thus be able to select a final modulus or rule, mainly with reference to its most desirable length, — no longer trammelled by the compounding of binary and tenary divisions. Ten million metres made the quadrant. Our octonary scale is also fur-

* "The new metrology of France, after trying it [the principle of decimal division] in its most universal theoretical application, has been compelled to renounce it for all the measures of astronomy, geography, navigation, time, the circle, and the sphere; to modify it even for superficial and cubical linear measure, and to compound with vulgar fractions, in the most ordinary and daily uses of all its weights and all its measures." (*Adams's Report.*)

nished with its grand unit (the eighth power of the octade,) which for want of a better name, we have christened *unkaly* [L,0000,0000]. The sextant of the equator is 21,911199$\frac{1}{3}$ feet or 262,934392 inches. This divided octavally into *unkaly* parts, gives us the quotient 15$\frac{2}{8}$ inches; *almost exactly our ideal measure!* Midway between the two great rival standards of olden time, the cubit and the foot,* it seems the very compromise of differences, the harmoniser of conflicting systems, and supplies us with a "module" perfectly suited to every requirement of popular mensuration. It needs but the application of octonary multiples, to complete a metrology simple and unexceptionable.

Before giving the table, however, we think it proper to suggest a slight modification in the divisions of the circle, as a subject controlling, to some extent, the details of our linear measures. Should the degree retain its present value as the 360th part of the circle, we should advocate strongly the employment of this unit of the equatorial circle, as the origin of our new standard of measure. Dividing the degree into *undersen* parts (L00,0000) we should have a module about one inch longer than that previously obtained, and somewhat nearer, therefore, to the ancient cubit. Its exact length would be 16.717 inches.

The number 60, however, approaches so near to the octonary *under*, (64,) that the temptation would be very strong to reduce degrees, minutes, and seconds, to the simplicity of the general notation, unless there appeared some strong reason for retaining the present sexagenary scale. But there is no special occasion for dividing small arcs into thirds or sixths, that gives this ancient and venerable system any advantage comparable to that we should have of adding up, or subtracting degrees, minutes, and seconds by a single operation, instead of resorting as now to reduction. On the contrary, the need of frequent binal division is here, as with other values, very apparent; and in this respect the number 60 is

* As the Milan foot is 15.62 inches, (very nearly the precise length above derived) and as the ancient cubits varied no less from the average standard, that of ancient Rome being 17.46 inches, while the *covid* of China was between 14 and 15 inches, our new standard might be called with propriety either "cubit" or "foot." If either of these designations is to be accepted, we shonld certainly prefer the "cubit."

very defective, as it permits but two bisections. The mariner's compass affords us a good illustration of the convenience experienced in a continued bisection of angles.* There would therefore be a positive benefit in substituting the number 64 for 60. This would interpolate 4 degrees into the sextant, or 6 degrees into the quadrant; making the right angle to be expressed by 96°, instead of 90°, as at present. This, then, is the table we should propose; in which it will be seen, the present values of arc are not so altered as to disturb appreciably our long established ideas of degree, minute, and second.

Divisions of the Circle.

LOO	(64)	tertials	make	1 second	=	0″. 823974
LOO	(64)	seconds	"	1 minute	=	52″, 734375
LOO	(64)	minutes	"	1 degree	=	$(\frac{15}{16})^{\circ}$ or 56′ 15″
LOO	(64)	degrees	"	1 sextant	=	60°
LPO	(96)	degrees	"	1 quadrant	=	90°
ЄOO	(192)	degrees	"	the semicircle	=	180°
БOO	(384)	degrees	"	the circle	=	360°

One obvious advantage of this scale, in addition to its simplification, would be to bring the azimuth compass into harmony with the mariner's compass, by giving them common measures. As the latter divides the quadrant into 8 "points" or "rhumbs," each of these would be LP° (12 degrees) instead 11° 15′, as at present; and the quarter-point would be Є° (3 degrees) instead of 2° 48′ 45″.

The zodiac, or ecliptic circle has from time immemorial been divided into twelve "signs." This would be found a very convenient unit to be applied to such arcs generally; as would also the smaller unit of its quarter, or LO°, the eighth part of the sextant. As there is no name for this, let us give it the name of "arc," (made technical and specific,) a name not inappropriate,

* The cardinal points dividing the circle into quarters, — each quadrant is divided into halves or octants, — each octant into halves and quarters, called "rhumbs" or "points," (8 in the quadrant,) — and finally, each of these points into halves and quarters; the rhumb or point being 11° 15′, and the quarter rhumb or point, 2° 48′ 45″.

since it is about the smallest arc we can readily distinguish from a straight line. This would give us the following scale :

L0° (8 degrees)	=	1 arc	=	7° 30′
P0° or 4 arcs	=	1 sign	=	30°
B0° or 6 arcs	=	1 octant	=	45°
L00° or L0 arcs or 2 signs,	=	1 sextant	=	60°
LP0° or LP arcs or 3 signs,	=	1 quadrant	=	90°

Should the above scheme of graduation for the circle be accepted, it will give an admirable simplicity to our table of lengths, which without further preface is herewith subjoined : —

Table of Linear Measure.

		1 point	=	($\frac{1}{260}$ inch nearly)	0.0038	ins.
L0 (8) points	make	1 line	=	($\frac{1}{33}$ inch nearly)	0.03	"
L0 (8) lines	"	1 nail	=	($\frac{1}{4}$ inch nearly)	0.245	"
L0 (8) nails	"	1 inch	=	(2 inches nearly)	1.959	"
L0 (8) inches	"	1 MODULE	=	($15\frac{2}{3}$ inches nearly)	1 ft. 3.672	"
L0 (8) MODULÈS	"	1 rod, or pole	=	3 yds.	1 ft. 5.37	"
L0 (8) rods	"	1 perch	=	27 yds.	2 ft. 7.	"
L0 (8) perches	"	1 furlong	=	222 yds.	2 ft. 8.	"
L0 (8) furlongs	"	1 mile	=	1 mile, 23 yds.	0 ft. 5.	"
L0 (8) miles	"	1 league	=	8 miles, 185 yds.	0 ft. 3.	"
L0 (8) leagues	"	1 *degree*	=	64 miles, 1480 yds.	2 ft. 5.	"
L0 (8) *degrees*	"	1 *arc*	=	518 miles, 1286 yds.	1 ft. 11.	"
L0 (8) *arcs*	"	1 *sextant*	=	4,149 miles, 1493 yds.	0 ft. 4.	"
6 (6) *sextants*	"	the *circumference*	=	24,899 miles, 158 yds.	2 ft.	"

The table of lengths proper terminates with the league ; the denominations following being those of arc. From the derivation of the standard, however, they coincide with precise measures, and are therefore properly included in the table. The " point " gives a dimension about that of the section of a human hair, or of a very fine grain of sand, and may be considered about the limit of visible magnitude. It is therefore a very suitable origin of linear value, while it is an equally appropriate point of departure for microscopic measurements. The " nail," and the " inch " are not the best terms for designating the new values; and are retained merely

5

from the difficulty of replacing them. The "Module" would in all human probability continue to be called a "foot"; though it deserves a name more expressive of its standard character. While this new *metre* gives us one of the most convenient rules we could devise, it also supplies us with a highly useful ten-and-a-half-foot measuring pole. But the peculiar beauty of the new Module is, that it precisely corresponds with the *tertial* of the new degree. *Under* Modules make one second; — (the "perch";) — *Under* seconds make one minute; (the "mile") — *Under* minutes make one degree; and *Under* degrees — the Sextant. Or, progressing by the successive squares, — *Unty* Modules make the rod; *Under* Modules make the perch; *Unsen* Modules make the mile; *Unkaly* Modules make the Sextant.

As referred to the French measures, we have for the value of our principal new denominations the following: the "line" = 0.77746 *millimetres;* the "nail" = 6.21975 *millimetres;* the "inch" = 4.9758 *centimetres;* the "module" = 3.98064 *decimetres;* the "rod" = 3.18451 *metres;* the "perch" = 2.54761 *decametres;* the "furlong" = 2.03809 *hectometres;* the "mile" = 1.63047 *kilometres;* and the "league" = 1.30437 *myriametres.*

For those measures in most common use, that is for those clustering immediately around the Module, it would doubtless be found highly convenient to give denominations to the halves and quarters; and thus conform them to the universal popular tendency to binary divisions. We therefore propose the following supplementary table; — not to be on any account incorporated with the preceding, nor in any respect to modify it; but to retain always its subordinate character.

2 nails	make	1 digit,	=	0.48975	ins.
2 digits	"	1 joint,	=	0.9795	"
2 joints	"	1 inch,	=	1.959	"
2 inches	"	1 hand,	=	3.918	"
2 hands	"	1 span,	=	7.836	"
2 spans	"	1 Module,	=	15.672	"
2 Modules	"	1 ell,	=	2 ft. 7.344	"
2 ells	"	1 fathom,	=	5 ft. 2.683	"
2 fathoms	"	1 rod, or pole,	=	10 ft. 5.376	"

Our tables of area, or of surface measure, would of course be derived directly from our linear measures, by the familiar law of squares.

Table of Square Measure.

LO (8) inches square,	or LOO (64) square inches,	make	1 square Module:
LO (8) Modules square,	or LOO (64) square Modules,	"	1 square rod:
LO (8) rods square,	or LOO (64) square rods,	"	1 square perch:
LO (8) perches square,	or LOO (64) square perches,	"	1 square furlong:
LO (8) furlongs square,	or LOO (64) square furlongs,	"	1 square mile.

For popular purposes, however, it would be necessary, or convenient, to have more numerous denominations of area measure; and a less rapid progression than that of *unders*, given in the above merely geometrical table of perfect squares. We therefore propose to insert intermediate values, so as to give our table the systematic or octonary form.

Table of Area — or Surface Measure.

		1 sq. Module	=	1 ft. 101.615 ins
LO (8) square Modules,	make	1 *sheet*,	=	1 yd. 4 ft. 92.92 "
LO (8) *sheets*,	"	1 sq. rod,	=	12 yds. 1 ft. 23.364 "
LO (8) square rods,	"	1 *plat*,	=	97 yds. 0 ft. 42.9 "
LO (8) *plats*,	"	1 sq. perch,	=	776 yds. 2 ft. 55.3 "
LO (8) square perches,	"	1 *acre*,	=	1 acre, 1370 yds. 1 ft. 10.42 "
LO (8) *acres*,	"	1 sq. furlong,	=	10 acres, 1280 yds. 8 ft. 83.4 "
LO (8) square furlongs,	"	1 *district*,	=	82 acres, 567 yds. 5 ft. 91.27 "
LO (8) *districts*,	"	1 sq. mile,	=	1 mile, 16 acres, 4541 yds. 0 ft. 10. "

The intermediate (alternate) denominations of this table are not *perfect squares;* hence it was thought more correct to assign terms to them indicative of their superficial character without the use of the prefix "square." We observe here one advantage that would result from the radix of numeration being a perfect square. The square root of 8, is 2.828427124; or √LO,=C.6POLLBLP6; hence this value would represent in any given units, the side of a square, equal to LO (8) of the square units. Thus the side of a square "sheet" would be C *Modules*, 6 *inches*, P *nails*, O *lines*, and L *point*. The side of a square "plat" would be C *rods*, 6 *Modules*, P *inches*, O *nails*, L *line*, and L *point*. The side of a square "acre"

would be C *perches*, 6 *rods*, P *Modules*, 0 *inches*, L *nail*, L *line*, and 8 *points*. And the side of a square "district" would be C *furlongs*, 6 *perches*, P *rods*, 0 *Modules*, L *inch*, L *nail*, 8 *lines*, and L *point*. A very simple parallelogram is however afforded us, which gives with precision the dimensions of these respective areas. Thus a "district," as a land measure, is a rectangular space of ground, measuring two furlongs in one direction, and four furlongs in the other; an "acre" a similar space of ground, measuring two perches in one direction, by four perches in the other; a "plat," a space measuring two rods in one direction, by four rods in the other; and a "sheet" is in like manner a surface of two Modules by four Modules. This table presents, therefore, the simplest ratios of superficial measure which could be devised; and would be found admirably adapted to every purpose of mensuration. For smaller surfaces, it is probable that the following supplementary table would prove a useful resort:

4	square	nails	make	1	square	digit	=	0.239858	ins.
4	sq.	digits	"	1	sq.	joint	=	0.959433	"
4	sq.	joints	"	1	sq.	inch	=	3.837735	"
4	sq.	inches	"	1	sq.	hand	=	15.350941	"
4	sq.	hands	"	1	sq.	span	=	61.403766	"
4	sq.	spans	"	1	sq.	Module	=	1 ft. 101.615	"
4	sq.	Modules	"	1	sq.	ell	=	6 ft. 118.460	"
4	sq.	ells	"	1	sq.	fathom	=	3 yds. 0 ft. 41.841	"
4	sq.	fathoms	"	1	sq.	rod	=	12 yds. 1 ft. 23.364	"

For measuring volume, we would naturally employ simply the cubes of the preceding denominations; while the contents of such cubic meters respectively, of distilled water at its maximum density, would as obviously furnish the denominations of weight. Throughout these derivative tables, we propose to adopt the MODULE as the universal standard. In this respect we perceive another great superiority in our linear unit, over that of the French system. From the inconvenient size of the *Metre*, it has been made practically a standard only of lengths. The *Are*, (the unit of surface,) is derived, not directly from the *Metre*, but from the *Decametre;* the *Litre* (the unit of capacity) is derived from the

cube of the *Decimetre;* and lastly, the *Gramme* (the unit of weight) is derived from the cube of the *Centimetre.* The contrasted simplicity of our project needs no comment.

Table of Volumes.

		1 cubic nail	=	0.01468 cubic inch.
10 (8) cubic nails	make	1 cub. digit	=	0.11747 cub. in.
10 (8) cub. digits	"	1 cub. joint	=	0.93977 cub. in.
10 (8) cub. joints	"	1 cub. inch	=	7.51817 cub. ins.
10 (8) cub. inches	"	1 cub. hand	=	60.14537 cub. ins.
10 (8) cub. hands	"	1 cub. span	=	481.16296 cub. ins.
10 (8) cub. spans	"	1 cub. MODULE	=	2.22760 cub. feet.
10 (8) cub. MODULES	"	1 cub. ell	=	17.82085 cub. feet.
10 (8) cub. ells	"	1 cub. fathom	=	142.56680 cub. feet.
10 (8) cub. fathoms	"	1 cub. rod	=	1140.53441 cub. feet.

This simple scale of volumes or bulks, derived directly from our smaller linear table, gives a good illustration of the great beauty and convenience flowing out of the employment of a radix of numeration which is a perfect cube. Each of the cubic measures of the above table has for the dimensions of its side 2 of the linear values above it.

Table of Derivative Measures.

The cubic nail	gives	the	*morsel*	measure,	and	the	*grain*	weight.
The cubic digit	"	"	*ligule*	"	&	"	*scrap*	"
The cubic joint	"	"	*cup*	"	&	"	*dram*	"
The cubic inch	"	"	*gill*	"	&	"	*ounce*	"
The cubic hand	"	"	*pint*	"	&	"	*pound*	"
The cubic span	"	"	*gallon*	"	&	"	*stone*	"
The cubic MODULE	"	"	MODIUS	"	&	"	PONDUS	"
The cubic ell	"	"	*but*	"	&	"	*load*	"
The cubic fathom	"	"	*cord*	"	&	"	*ton*	"
The cubic rod	"	"	*vat*	"	&	"	*keel*	"

This table furnishes us with a complete system. It needs but a simple calculation to exhibit our weights and measures in full.

Our measures of capacity, with their respective values are as follows.

Table of Capacity Measure.

		1 minim	=	.488	min.
10 (8) minims	make	1 morsel	=	3.905	"
10 (8) morsels	"	1 ligule	=	31.244	"
10 (8) ligules	"	1 cup	=	4 drs. 9.955	"
10 (8) cups	"	1 gill	=	4 oz. 1 dr. 19.64	"
10 (8) gills	"	1 pint	=	2 pts. 1 oz. 2 drs. 37.	"
10 (8) pints	"	1 gallon	=	2 galls. 0 pts. 10 oz. 4 drs. 56.	"
10 (8) gallons	"	1 MODIUS, or bushel,	=	16 galls. 5 pts. 4 oz. 7 drs. 35.	"
10 (8) MODIUSES	"	1 butt	=	133 galls. 2 pts. 7 oz. 4 drs. 44.	"
10 (8) butts	"	1 cord,	=	1066 galls. 3 pts. 12 oz. 5 drs. 56.	"
10 (8) cords	"	1 vat,	=	8531 galls. 6 pts. 5 oz. 7 drs. 28.	"

Our language is unfortunately but very poorly supplied with terms expressive of capacity; and as the existing names for the smaller liquid measures used by the apothecary ("fluid-drachm," and "fluid-ounce,") are exceedingly objectionable, from their reference to the incongruous standard of weight, — we are compelled to reject them, although we have no appropriate denominations to substitute. The word "morsel" is perhaps sufficiently indeterminate to answer the purpose; and the Roman *ligula*, a small measure of about a spoonful, supplies a convenient term, having the same recommendation. The "cup," which is equally indefinite, represents about a half-ounce. The *Modius* of the Romans was about the quarter of a bushel: the term has been selected as a suitable one for indicating a standard *measure*, and also as suggesting its dimension, as the cube of the Module. The circumstance that it is here applied to a much larger volume than it was originally, is comparatively unimportant. The "cord" may very well be used in an extended application, — in the absence of any more suitable term that can readily be suggested. It expresses here a volume about one-tenth greater than the cord used as a wood measure.

As referred to our common table of "dry measure," as it is called, the new "pint" is equal to 1.79 pints: the new "gallon" is equal to 7 quarts and one-third of a pint; — or about one-twelfth

less than a peck: the new "Modius" — or "bushel," contains 3849.3 cubic inches; and is therefore equal to one bushel, 3 pecks, 1 quart, and one-half a pint, — or to very nearly $1\frac{3}{4}$ bushels, — the U. S. bushel containing 2150.4 cubic inches: the new "butt" is equal to 14 bushels, 1 peck, 2 quarts, and half a pint, (being about one-fourth more than the English butt of 2 hogsheads;) and the new "cord" is equal to 114 bushels, 2 pecks and 2 quarts.

In the French measures, our "pint" is very nearly equal to the *litre*, — being .9855 of a *litre;* our "gallon" = 7.884 *litres;* and our "Modius" = 63 *litres.*

It may not be out of place to mention here, (as exhibiting an interesting and very early anticipation of our octonary scale of measures, in England,) that by the act of 51st. Henry III, — (1266) it was declared that "8 pounds [of wheat] do make the gallon of wine, and 8 gallons of wine do make a London bushel, and 8 London bushels do make the quarter."

Our proposed system of weights forms but a corollary from the preceding table of capacity measures; — a Modius of pure water forming the standard unit, — which we therefore call our *Weight* or *Pondus.* By the best recent determinations, one cubic inch of distilled water at maximum density, (or $39\frac{1}{4}$ degrees, Fahrenheit,) weighs 252.8 grains Troy.* The Modius or cubic Module would therefore weigh 973104.697 grains; or 139 pounds avoirdupois, and 104.7 grains. This will give us the following table:

Table of Weights.

		1 mite	=			0.464	grs.
L0 (8) mites	make	1 grain	=			3.712	"
L0 (8) grains	"	1 scrap, or scruple,	=			29.696	"
L0 (8) scraps	"	1 dram	=	(avoirdupois)	$\frac{1}{2}$ oz.	18.824	"
L0 (8) drams	"	1 ounce	=		4 oz.	150.595	"
L0 (8) ounces	"	1 libra, or pound,	=	2 lbs.	2 oz.	329.76	"
L0 (8) libras	"	1 stone	=	17 lbs.	6 oz.	13.08	"
L0 (8) stones	"	1 PONDUS, or Weight,	=	139 lbs.		104.7	"
L0 (8) PONDUSES	"	1 load	=	1112 lbs.	1 oz.	400.	"
L0 (8) loads	"	1 ton	=	8896 lbs.	15 oz.	138.	"
L0 (8) tons	"	1 keel	=	71175 lbs.	10 oz.	220.	"

* This is a trifle more than the value adopted by Mr. Hassler, for the U. S. standards; his experiments giving 252.745 grains to the cubic inch.

While the "Pondus" is the standard of determination, the pound, as the unit of weight in most common use, would be the secondary or derivative standard. Since *Under* "pounds" make the "Weight," this corresponds to our present hundred weight. To prevent any confounding of the "Pondus" and its derivative "pound," we should prefer the restoration of the old word "*libra.*" The "load" is not quite half a ton: and the new "ton" is very nearly 4 tons.

The "keel" is one [redacted] half [redacted] larger than the English keel (a weight used only for coal,) which is equal to 21 tons, 4 cwt.; and of which twenty make a "ship-load." Or the English keel is two-thirds of our "keel" as above given.

Estimated by the French weights, our "scrap" = 1.924 *grammes;* our "dram" = 1.5393 *decagrammes;* our "ounce = 1.2314 *hectogrammes;* and our "libra" or pound = .98514 *kilogrammes.*

It would probably be found convenient to distribute the more popular or frequently used weights (those from the "scrap" or "scruple," to the "libra,") upon the binary scale: but as the divisions of halves and quarters practically accomplish this, it seems hardly necessary to suggest a series of intermediate denominations.

An entire remodelling of our coinage would of course be necessary under our octonary system. That such a remodelling is really very much needed, — notwithstanding the vaunted excellence of our currency, and its real superiority to that of almost every other nation, — may, we think, be very clearly shown. The universal prevalence of binary divisions rendered necessary by the wants of trade in all its departments, and the signal inability of the established system of coinage to meet such want, has already been alluded to in an early part of this Report. As specie is merely the representative of value, the proportions of it required in exchange for commodities, must of course be determined by the necessary or convenient divisions of the commodity, and not by the size of the pieces which make the money. If the two are incommensurable, a sacrifice is demanded. Of the smaller articles usually sold in packages, a dozen, forms the most common measure; but we are aware of no single article being usually put up by tens

in correspondence with the coins which are to purchase them. Even articles of furniture, such as chairs, plates, cutlery, &c., are generally sold by the dozen, — but never by the decade.

In the subdivision of articles, either by weight or measure, the universal requirement is that of *halves* and *quarters*; — never that of fifths or tenths. Hence it results of necessity, that prices must have the same bi-sections; without any reference to the arbitrary and oppressive though impotent exactions of a decimal regency. If we select any familiar article sold by the druggist,— for example, bicarbonate of soda or tapioca, worth a quarter of a dollar a pound, — it is plain that in the present state of our coinage, it is impossible for a purchaser to pay for half a pound. But the half pound must be had, and so it must be paid for either by 12 cents or by 13. The continued bi-sections of our dollar, actually required in the business of the shop in almost every branch of trade, give us $12\frac{1}{2}$ cents for the eighths, $6\frac{1}{4}$ cents for the sixteenths, and $3\frac{1}{8}$ cents for the thirty-seconds. As none of these fractions are in use, they are necessarily sacrificed on the one side or the other, — and by no means in a way to equalize the exaction; for it is a settled principle of finance with some establishments — both large and small, never to lose the half-cent; nay, even to secure the over-charge of three-quarters of a cent, rather than surrender the quarter. So that by the popular arithmetic of trade as taught by those who seem to be most "knowing," we learn that the subtraction of six from twenty-five, leaves eighteen as a remainder.

The gradual withdrawal of the Spanish and Mexican eighths and sixteenths of our dollar from circulation, has been productive of much inconvenience; * and has of course done nothing towards

* Mr. Adams, writing under a different state of things, when half cents were common, and Spanish "fips" and "levies" abundant, remarks: "These eighths and sixteenths form a very considerable proportion of our metallic currency: and although the eighth dividing the cent only into halves, adapts itself without inconvenience to the system, the fraction of the sixteenth is not so tractable; and in its circulation, as small change, it passes for six cents, though its value is six and a quarter, and there is a loss by its circulation of four per cent. between the buyer and the seller. For all the transactions of retail trade, the eighth and sixteenth of a dollar are among the most useful and convenient of our coins; and although we have never coined them ourselves, we should have felt the want of them, if they had not been supplied to us from the coinage of Spain." (*Report to Congress on Weights and Measures.*)

effecting the substitution of our native dimes and half-dimes, as had been vainly imagined by some. Our current prices seem to now embrace almost every figure *except* the decimals; 3 cts., 6 cts., 9 cts., 12 cts., 13 cts., 18 cts. and 19 cts, are the common rates below the quarter of a dollar; and above it we have 31, 37, 38, 44 (sometimes shaved down to 43,) 56, 62, 63, 69, 75, 81, 87, 88, and 94 cents; but *never* the 30 cts., 40 cts., 60 cts., 80 cts., or 90 cts., decreed for us by our legislators. In view of these incongruous conditions, it cannot require argument to recommend the simple Money Table we here propose.

Money Table.

Denominations.				Dollar value.	Proposed value.
		1 penny	=	.01$\frac{9}{16}$ cts.	.01$\frac{7}{8}$ cts.
LO (8) pennies	make	1 groat or bit	=	.12$\frac{1}{2}$ cts.	.15 cts.
LO (8) groats	"	1 dollar	=	$1.00	1.19\frac{9}{10}$
LO (8) dollars	"	1 real	=	$8.00	9.59\frac{1}{5}$
LO (8) reals	"	1 eagle	=	$64.00	76.73\frac{3}{5}$

(*Under* "pennies" make the "dollar"; *under* "dollars" make the "eagle.") The middle column of the table represents the respective value of each denomination on the assumption of the present dollar as the standard. Our scheme contemplates, however, a much more radical improvement than this; great as this alone, would undoubtedly be. The broom of reform must sweep clean; and we must not stop so long as there appears the possibility of a further improvement. The trouble and inconvenience of a transition is no greater to a perfect system than it would be to an imperfect one.

Accordingly we would have in the first place, the standard of purity elevated, and made permanent; and have it expressed in octonary terms. We would have the mint standard fixed at 22$\frac{1}{2}$ carats fine, to 1$\frac{1}{2}$ carat alloy, both for gold and silver:—that is, $\frac{15}{16}$ of pure gold, or of pure silver, to $\frac{1}{16}$ of base metal,—or in our octonary values,—*Kity fo* (BF) parts pure, to *fo* (F) parts alloy,

in *under* (LOO) parts of coined metal. This standard we would have made *invariable,* — placed if possible under the protection of the constitution, — to be changed or debased on no pretence.

In the next place, we would make *gold* the permanent standard of value ; and would make its coins exact weights, and measuring exact lengths ; — these also of course to be permanent and invariable. The silver coins would fluctuate slightly in value, according to the current rate of the metal as compared with gold. To preserve their denominational values, the weights of the coins might be adjusted every octade, or eight years : — the diameters of the coins, however, not to be altered, unless the comparative values of the two metals should change considerably : — the ordinary adjustment of weight being effected by modifying the thickness merely of the coins. Our highest coin — the "eagle" (new style) we would make to weigh exactly 8 "drams," or 1 "ounce," (new style ;) and its diameter exactly 8 " nails," or 1 " inch," (new style,) — as we find this would give a coin of nearly the same proportions (or relation of thickness to diameter) as our present eagle. Of the 8 " drams," one half a dram or 4 "scraps" would be (as before indicated) alloy ; the remaining $7\frac{1}{2}$ drams being pure gold. Such a coin we find by calculation would be worth \$76.73$\frac{5}{8}$ cts. Hence is derived the second series of values, found in the last column of the above table.

To preserve the same proportions of diameter to thickness in all the gold coins, our octonary scale gives us a very simple ratio ; for as the diameters of the coins would be as the cube roots of their bulks or weights,— it results that a gold coin eight times as heavy as the eagle should have exactly twice its diameter ; or on the other hand, that one an eighth of its weight should have exactly one half its diameter. The "real" is such a coin, and would therefore be exactly 4 " nails" in diameter, if its thickness were also one half that of the " eagle." Here we perceive once more the beauty and convenience of having the radix of numeration, a perfect cube ; as is our *unty* (LO). One very great advantage of the system here proposed, is that it would supply the people with a scale of highly useful practical weights, and with a constant

reference to original or national standards. Another benefit, no less striking, is that it would give them on the other hand, the most convenient means of determining the genuineness of the currency, and thus provide an increased security from the impositions of the counterfeiter.

The silver coins should obviously be on a different scale of size; so that no two coins should have the same diameter. A very suitable size for the "dollar" would be 6 "nails," and of course in like manner the "groat" would be 3 "nails." In our present coinage silver is just 16 times heavier than gold, the alloys being the same and the values the same. This would make our new "dollar" weigh 2 "drams" (new style.) But as this weight could not be permanently maintained, (the barometric scale of silver being, as before remarked, liable to fluctuations,) we shall mark it with a "more or less."

On an octonary scale, all the subordinate coins required for representing all possible values, would be merely halves and quarters. Here, again, we perceive another beauty in our system, that it gives the maximum range of expression, with the minimum number of pieces. As we have five different coins in our table, a half and quarter to each would give us fifteen pieces, of which 6 would be in gold, 6 would be in silver, and 3 would be in copper. The "dollar" should be coined only in silver. We here have a distribution of moneys on a purely binary scale, (that scale which has been found to be of all others the most beautiful, and the most convenient) and running through a range of value considerably greater than is to be found in any nation — a range of binary progression perfect and unbroken, from the farthing to the sixteen pounds sterling. A more simple arrangement, or one more suited to the popular necessities, and more easy of popular comprehension and acquisition, cannot be devised. With these preliminary conditions and determinations, we are prepared to give the following table of our proposed coinage:

Table of Coins.

	Coins.	Size.	Weight.	Value.
Copper.	Quarter-penny	2 nails, 2 lines		$\frac{15}{32}$ of a cent
	Half-penny	2 nails, 6 lines		$\frac{15}{16}$ of a cent
	Penny	3 nails, 4 lines		.01$\frac{7}{8}$ cts.
Silver.	Quarter-groat.	1 nail, 7 lines	(4 grains, $\pm$)	.03$\frac{3}{4}$ cts.
	Half-groat	2 nails, 3 lines	(1 scrap, $\pm$)	.07$\frac{1}{2}$ "
	Groat	3 nails	(2 scraps, $\pm$)	.15 "
	Quarter-Dollar	3 nails, 6 lines	(4 scraps, $\pm$)	.29$\frac{39}{40}$ "
	Half-dollar	4 nails, 6 lines	(1 dram, $\pm$)	.59$\frac{19}{20}$ "
	Dollar	6 nails	(2 drams, $\pm$)	1.19\frac{9}{10}$ "
Gold.	Quarter-real	2 nails, 4 lines	2 scraps	2.39\frac{4}{5}$
	Half-real	3 nails, 2 lines	4 scraps	4.79\frac{3}{5}$
	Real	4 nails	1 dram	9.59\frac{1}{5}$
	Quarter-eagle	5 nails	2 drams	19.18\frac{2}{5}$
	Half-eagle	6 nails, 4 lines	4 drams	38.36\frac{4}{5}$
	Eagle	8 nails	1 ounce	76.73\frac{3}{5}$

Our new "penny" is very nearly equal to the English penny; having [illegible] of its value;—that is, our new "dollar" is just equal to 60 English pennies, instead of the 64 pennies of our table. The French value of our "penny" is 10$\frac{5}{32}$ centimes. Our new "groat" is about double the English groat, being 7[illegible]*d.* or 81$\frac{1}{4}$ centimes, French. Our new "dollar" is equal to 5*s.* [illegible]*d.* English; or 6 francs, 50 centimes, French. Our new "real" is equal to 2£. [illegible] English; or 52 francs (2 napoleons, 12 francs,) French. And our new "Eagle" is equal to 16£. [illegible] English; or 416 francs (20 napoleons, 16 francs,) French.

We shall here present the monetary system suggested by Mr. Adams, in his able Report on Weights and Measures, to which we have been so much indebted, in order that the great superiority of the plan above detailed wherein it differs from that of Mr. Adams may be made apparent, and that the weight and authority of his name may be adduced wherein the projects agree.

"If it were the duty of this Report to present a system of

weights, measures, and coins, all referable to a single standard, combining with it, as far as possible, the decimal arithmetic, and of which uniformity should be the pervading principle, without regard to existing usages, it would propose a silver coin of nine parts pure and one of alloy: of thickness equal to one-tenth part of its diameter; the diameter to be one-tenth part of a foot, and the foot one fourth part of the French metre. This dollar should be the unit of weights, as well as of coins and of accounts; and all its divisions and multiples should be decimal. The unit of measures of capacity should be a vessel containing the weight of ten dollars, of distilled water, at the temperature of ten degrees of the centigrade thermometer; and the cubical dimensions of this vessel should be ascertained by the weight of its contents; the decimal arithmetic should apply to its weight, and convenient vulgar fractions to its cubical measure. This system once established, the standard weight and purity of the coin should be made an article of the constitution, and declared unalterable by the legislature. The advantage of such a system would be to embrace and establish a principle of uniformity with reference to time which the French metrology does not possess. The weight would be a perpetual guard upon the purity and value of the coin. No second weight would be necessary or desirable. The coin and the weight would be mutual standards for each other; accessible at all times to every individual. Should the effect of such a system only be, as its tendency certainly would be, to deprive the legislative authority of the power to debase the coins, it would cut up by the roots one of the most pernicious practices that ever afflicted man in civil society. By its connection of the linear standard with the French metre, it would possess all the advantages of having that for a unit of its measures of length, and a link of the most useful uniformity with the whole French metrology."

When we consider the decided testimony Mr. Adams has borne against the fitness and utility of a decimal metrology, and the clear manner in which he has himself presented its disadvantages, we must regard the scheme he has here suggested, rather as a reluctant assent to a choice of evils, than as the matured and satisfactory determination of his deliberate judgment. In our opinion, a

reference to the French *metre*, as a standard, would be in almost every respect objectionable, unless we accepted with it the entire metrical system. The fourth part of the metre, or the ten-inch rule, we regard as a much less convenient popular measure, than would be its half, or the twenty-inch rule: as we have already expressed our conviction that the cubit of 18 inches comes probably nearer the average of a maximum utility, than any other of the scales which have hitherto been in common use. Nor do we believe that the system itself, in its present form possesses the elements of a general ascendency, or even of a permanent establishment. When in a succeeding generation, the admiration naturally excited by a new and beautiful mechanism shall have subsided to a more impartial and experienced estimate of its defects, as well as of its admirable features, it will, doubtless, through the inevitable laws of human nature, undergo many and very important modifications, unless indeed some phœnix from its ashes shall arise and be judged worthy to replace it.

But there is an objection even stronger than any we have yet noticed, to accepting the metre, either directly or indirectly, as our national standard of length; and that is the want of absolute precision in the rule itself. It has been shown by the investigations of able mathematicians, that in consequence of a slight oversight in an early stage of the tedious and complicated reductions required, (unfortunately not detected until after the complete organization and establishment of the national standards of weight and measure,) the metre is not an exact expression of its theoretical value.* The

* "It appears that through inadvertence, or the misapprehension of a formula, the distance between the parallels of Montjouy and Mola (the station on Formentera) was computed without applying the proper correction for the convergence of meridians. The distance originally given by the commissioners was 153605.77 *toises;* but the distance when the computation is correctly made is 153673.61 *toises.* (*Bessel's Astronomische Nachrichten*, No. 438,) the difference being 67.84 *toises*, [about 434 feet.] In consequence of this correction, and of the introduction of some other arcs since measured, into the data for determining the figure of the earth, the length of the quadrant of the meridian expressed in terms of the legal *metre*, according to the best determination which can at present be made, of the dimensions of the earth, is 10,000855.76 *metres;* in other words the legal *metre* would require to be lengthened by about the .00000856 part of itself in order to agree with the ideal *metre*, a ten millionth part of the quadrant of the elliptic meridian." (*Penny Cyclopedia*, vol. 25, page 216,—article, "*Trigonometrical Survey.*")

distance from the pole to the equator on a meridian line, was computed to be 5,130740 toises, (the toise being 6 $\frac{4}{10}$ feet, or more exactly 6.394219 feet,) and the length of the metre was definitely declared by the National Assembly, in 1799, to be 443.29596 lines of the *toise du Perou.* "Later and more extensive measurements," says Mr. Guyot, "in various parts of the globe, however, seem to indicate that this quantity is somewhat too small. The latest and most exact results we now possess, combined and computed by Bessel, would make the quarter of the meridian 10,000856 metres, and the metre = 443.29979 Paris lines: Schmidt's computation would make it 443.29977 lines, and both numbers are confirmed by Airy's results." (*Tables, &c., by A. Guyot, Smithsonian Publication.*) The metre is, therefore, too short by .00383 of a line; a deficiency which cannot be regarded as trivial. An error of $\frac{1}{30000}$ of an inch in 40, though wholly inappreciable to the dry-goods dealer, is to the astronomer an amount of serious import; and even to the refinements of our mechanical art, is a discrepancy of more than three times the quantity which can be distinctly measured.* This unfortunate and vital defect in the French metre nullifies almost entirely its value as a natural standard; and defeats that principal object of its establishment, — the facility of its perfect restoration in all future time, should the existing material standards be destroyed.

In the new standard of length, proposed and developed in this Report, we believe that every excellence of the French standard has been carefully preserved, and all its imperfections as successfully avoided. Starting from the same general principles by which that was obtained, we have made no departure from the details of its derivation, not required by the plainest and soundest deductions of experience, philosophy and common sense. Does the French method propose an *aggravated* yard as a convenient unit,

* We learn from the department of "Weights and Measures," at Washington, under the Coast Survey Bureau of the United States, that the hundred-thousandth of an inch is a measure of accuracy easily recognized by the delicate instruments introduced by the ingenious mechanical artist, Mr. Joseph Saxton, who is in charge of this department; and that even this limitation is imposed solely by the uncertainties of the temperature correction.

we show the superiority of the cubit. Does it (on good grounds, at the time) select an elliptical meridian, as its origin of measure, we show still better grounds for preferring the equatorial circle. Does it look (almost necessarily) to the quadrant as a natural unit, we show the greater propriety of the sextant. Does it rest on a thoroughly decimal basis, we show the most cogent reasons for adopting an octonary distribution. Does it find a fitting divisor only in the seventh power of its decimal radix, — we accidentally find it in a great arithmetical unit — the eighth power of the octade. Does it finally give as its finished product, an imperfect *metre*, we offer for acceptance a perfect *Module*.

The system of metrology derived from this new standard, has in it nothing that is arbitrarily assumed. Each part of it is dependent upon every other, and each part flows from each, by a logical and systematic necessity. The whole is thus a perfect unit, simple and complete, — comprehending every relation of dimension and of weight, and adequate to every purpose of precision, the minutest as well as the grandest.

There remain one or two other applications of the octonary measure, however, which though incidental, are still of sufficient importance to merit a special notice. If we substitute for the sexagenary divisions of the circle, the scale of *unders*, a very obvious and pressing analogy would require the same substitution in the smaller divisions of *time*. The hour itself, — the twenty-fourth part of the day, is too convenient a unit to be lightly changed. The attempt made by the French metrology to decimalize it, was only a notable failure.* Nor is there any special reason for disturbing it here, since by the octonary notation, there are just *thety* (60) hours to the day. The minutes and seconds may, however, be changed without much inconvenience, and with manifest advantage. The substitution of the number 64 (L00) for 60, would make the minute $\frac{1}{16}$ shorter than it now is; and would give us

* "The solar day was divided into ten hours, each of 100 minutes, and each minute of 100 seconds. . . . . The division of the day into a hundred thousand parts, was the first part of the system that was abandoned. It had been decreed as compulsory with the new nomenclature of the calendar on the 24th of November, 1793, (4 *Frimaire* 2,) but this regulation was indefinitely suspended by the law of 7th April, 1795, (18 *Germinal* 3.)" — (*Adams's Report.*)

4096 (L,000) seconds to the hour, instead of the present number, 3600. This would, of course, reduce the length of the seconds' pendulum correspondingly. Assuming the length of the present seconds' pendulum to be 39.1393 inches, we have the proportion, $(4096)^2 : (3600) :: 39.1393 : 30.23417$. Therefore the length of the new seconds' pendulum, (or the octonary pendulum, as we may call it) is 30.23417 inches, or not quite 2 modules. This modification of the second, would give us the following table of time measure:

Time Table.

		1 second (n. s.)	=	$\frac{221}{256}$ of a sec:
L00 (64) seconds	make	1 minute (n. s.)	=	$\frac{15}{16}$ of a min: or 56.25 s.
L00 (64) minutes	"	1 hour	=	2 "arcs" of rotation.
60 (24) hours	"	1 mean solar day	=	1 rotation.
66 (30) days +	"	1 month	=	1 lunar revolution +.
LP (12) months	"	1 year	=	1 solar revolution.
L0 (8) years	"	1 octade	=	8 years.
L0 (8) octades	"	1 undery	=	64 (L00) years.

The value of the new style second, decimally expressed, is 0.86328125 sec. As there would be *unsen* (L,0000) seconds in one hour, and *thety* (60) hours in one day, there would of course be *thety-sen* (60,0000) seconds (98,304) in one day. The 365 days, 5 hours, 48 min. and 48 sec. which form the solar year,* would be in the new style PPP days, P hours, 6P min. and 6.6 sec. The new octonary "century" must have a name, let us call it "undery;" then instead of the "18 centuries" of the Christian era, we must say the "*thety-pa underies.*"

As the Barometer is graduated by absolute lengths, the only change involved in the reading of this instrument, would be the simple substitution of the Module divisions, for those of the foot. It would be found convenient, however, to read by "nails," rather than by larger quantities. Thus 30 inches (the average height of the barometer column) is L "Module," 8 "inches," C "nails,"

* According to Lacaille's tables, 365 days, 5 hours, 48 min. 49 sec. According to Bessel in 1830, 365 days, 5 hours, 48 min. 47.8 sec.

and F "lines;" which would be much simpler in the form of LBC.F "nails," or *under kity-du* "nails" and a half. The following table exhibits the barometric range by intervals of 4 "nails" (or one inch very nearly) from *under* (LOO) to *duder* (COO) "nails;" that is, from the height of one Module (the half of an atmosphere) to that of two Modules, (or a whole atmospheric pressure.) One inch = 4.0837 "nails;" F "nails" = 0.97948 inch.

Table of the Barometer Scale.

C00	(duder)	"nails"	=	31.344	inches	=	796.127	millimetres.
L8F	(under kity-fo)	"	=	30.364	"	=	771.248	"
L80	(under kity)	"	=	29.385	"	=	746.369	"
L6F	(under sety-fo)	"	=	28.405	"	=	721.490	"
L60	(under sety)	"	=	27.426	"	=	696.611	"
LPF	(under paty-fo)	"	=	26.446	"	=	671.732	"
LP0	(under paty)	"	=	25.467	"	=	646.852	"
LFF	(under foty-fo)	"	=	24.487	"	=	621.974	"
LF0	(under foty)	"	=	23.508	"	=	597.095	"
L6F	(under thety-fo)	"	=	22.528	"	=	572.216	"
L60	(under thety)	"	=	21.549	"	=	547.337	"
LCF	(under duty-fo)	"	=	20.569	"	=	522.458	"
LC0	(under duty)	"	=	19.590	"	=	497.579	"
LLF	(under unty-fo)	"	=	18.610	"	=	472.700	"
LL0	(under unty)	"	=	17.631	"	=	447.821	"
L0F	(under and fo)	"	=	16.651	"	=	422.942	"
L00	(under)	"	=	15.672	"	=	398.063	"

In the application of the octonary scale to the Thermometer, a greater change would be desirable. None of the existing graduations of this instrument can be regarded as being at all satisfactory. The pervading principle of the French metrology, naturally produced the Centigrade scale; * but outside of the decimal sys-

* The Centigrade scale appears really to have originated with Andre Celsius, a Swedish astronomer and mathematician who lived during the first half of the eighteenth century, (from 1701 to 1744.) It is hence very frequently designated as the "Celsius Thermometer." In addition to the scales of Fahrenheit, Reaumur, and Celsius, two others have been devised, which have never gone into use; those of De Lisle, and of Murray. De Lisle made the boiling point 0°, and counting downward made

tem, it has nothing to recommend it. Its degrees are of a less convenient size than those of the Fahrenheit scale; and hence the continued use of the latter in America, and in England, notwithstanding its awkward and arbitrary character.

The time appears now to have nearly arrived for a philosophical graduation of this most important instrument, from the *absolute zero*, so that its degrees shall accurately represent proportional increments of heat. Although this point of "no heat" cannot yet be determined with any precision, there are several considerations which induce the belief that it cannot be lower than 500 degrees below the zero of Fahrenheit. The conclusion arrived at by Clement and Desormes, is that the absolute zero is most probably at —266½° Cent. or at —448°, Fahr.* This would make (448° + 32°) 480° Fahr. below the freezing point; and 180° Fahr, being the interval between that point and the temperature of boiling water, we should have the boiling point by Fahrenheit's graduation, at 660° above the absolute zero. The objection which might be raised to the use of such large figures to express our ordinary temperatures, (as 560°, for the summer heat of 80°) might be easily obviated by the natural suggestion of making the melting point of ice, a high unit, such as *unty-der*, (1000) — so that being the commencement of a new count, it might thus be popularly accounted a zero point; — the *unty-der* unit being suppressed, or

the freezing point 150°. Each of these degrees was therefore $\frac{6}{5}$ of a degree of Fahrenheit. Murray selected the freezing point of Mercury, — 39° Fahrenheit, for his zero, and made its boiling point (661° Fahr.) to be expressed by 1000°. By this system, the point of melting ice would be $101\frac{3}{7}$°, and that of boiling water, $358\frac{4}{7}$°; each degree being $\frac{7}{10}$ of a degree Fahr.

* "Clement and Desormes place the absolute zero at—266°.6' C. (—447°.9' Fahr.) partly with reference to a not very probable hypothesis concerning the heat contained in a vacuum, partly from the following considerations: Air at 0° [the freezing point of water] expands for each degree C. of heat superadded, by $\frac{375}{100\,000}$ or $\frac{1}{266.6}$ and contracts for each degree of heat abstracted, by $\frac{1}{266.6}$. If this law be applicable at all temperatures, the limit of diminution of volume, must be found at—266.6; below this point there can be no further contraction of volume, and therefore no further abstraction of heat. Or, if 266.6° of heat be imparted to air at 0° C., its volume becomes doubled; according to the above law, the air when its volume is doubled, must contain twice the quantity of heat which it contains at 0°; consequently at 0°, it must contain 266.6° more heat than at the absolute zero." (*Gmelin's Hand-book of Chemistry; translated by Henry Watts, vol. 1, p. 303.*)

always understood. If we accept this figure experimentally, in order to ascertain the boiling point of such a scale, we have the simple proportion, 480° Fahr. : L000. (512) :: 180° Fahr. : 600. (192.) This gives us a very convenient number (*theder*) for our new boiling point, and the following scale results:

Table of Absolute Thermometer Degrees.

Absolute.	Fahrenh't.	Centigrade.	Reaumur.	Indications.
600	212	100	80	*Boiling point of water.* (Bar. 30 ins.)
C60	197	91⅔	73⅓	Meconine fuses. Scheererite boils.
CP0	182	83⅓	66⅔	Nitric Ether, Propione, & Metacetone, boil.
CC0	167	75	60	Absolute Alcohol, Camphrone & Acetal, boil.
C00	152	66⅔	53⅓	Water scalds. Wood spirit boils.
L60	137	58⅓	46⅔	Potassium fuses. Bromine boils.
LP0	122	50	40	Metal painful to the touch.
LC0	107	41⅔	33⅓	Fever heat. Spermaceti fuses.
L00	92	33⅓	26⅔	Highest (shade) temperature at London.
60	77	25	20	Mean temperature at Havana and Vera Cruz.
P0	62	16⅔	13⅓	Mean temperature at Rome; and at Mexico.
C0	47	8⅓	6⅔	Mean temperature at Gottingen & Edinburgh.
L000	32	0	0	*Melting point of ice.*
860	17	—8⅓	—6⅔	Oil of turp'tine freezes. Caoutchoucine melts.
8P0	2	—16⅔	—13⅓	Mean temperature at Melville Island.
8C0	—13	—25	—20	Bromine melts. Mean temperature at N. Pole.
800	—28	—33⅓	—26⅔	Lowest temperature at St. Petersburg.

Each degree Absolute = $\frac{15}{16}$ of a deg. Fahr. — or $\frac{25}{48}$ of a deg. Cent. — or $\frac{5}{12}$ of a deg. Reaum.

We have not thought it necessary to present the graduation of this scale by smaller intervals than *duty* degrees, (C0°) nor to extend it above the boiling point, as its rate of progression is sufficiently obvious. This scale is proposed merely as an illustration of the principle of graduation which we think should be adopted; and not, as in the case of the preceding tables, as being a final or permanent result. The estimates of different experimenters, are unfortunately, as yet, considerably discordant; and it will probably

require much more of fine experiment and careful induction, before a concurrence of considerations shall conspire to give us a reasonable confidence in the determination of the absolute zero.* Meanwhile the scale here suggested might be provisionally adopted with advantage, as superior to any of the existing scales, and as probably destined to require no very great modification on the attainment of a more complete knowledge of the subject.

We think that the melting point of ice should be selected as the first great unit or completed period, rather than the boiling point of water. First, because it is less dependent on barometric conditions than the latter; and secondly, because it lies much nearer our ordinary temperatures, and it is more convenient to reckon upward from a fixed point than it is to estimate downward. Should the absolute zero be ascertained to fall considerably below the — 500° of Fahrenheit, then we would recommend the employment of the next higher unit, the *unsen* (L,0000) to mark the melting point of ice, making the size of the degree to depend (within certain limits,) on the relation of these two points; and allowing the boiling point to fall wherever the arithmetical continuation of the scale should require; unless indeed, the adjustment of a degree or so, at the zero, (which would be of no practical consequence,) should be found to present the accommodation of a simple numerical relation, for the easy conversion of one scale into the other. The limiting conditions we would propose to the size of the degree, would be to select for the notation of the melting point of ice, that unit (of *un* and

* The estimate of Clement and Desormes (480° F. below freezing point,) is however probably near the truth, as it approximates tolerably well to the results derived by various other experimenters, from different data. Thus by Rudberg's experiments, confirmed by Magnus and Regnault, the absolute zero (of gaseous tension) is placed at — 273°.89 Cent., or — 461° Fahr. (493° F. below the melting point of ice.) By J. P. Joule, the absolute zero is placed at — 459° Fahr., (491° F. below the melting point of ice.) (*Philosophical Transactions of the Royal Society, London, for* 1852.) This would make our boiling point CBF° (188), or in other words would make our *theder* (600°) four degrees above the boiling point. Had the estimate of Clement and Desormes been 32 degrees lower — that is, had the 480° been below the *zero* of Fahrenheit's scale, instead of its freezing point, it would have made our octonary degrees exactly equal to the degrees of that scale, and perhaps for a provisional scale this would be a preferable notation, as it would involve no change in the graduation of our common Thermometer, but would require only a different reading; the melting point of ice being L000 instead of 32, and the boiling point CBF (180) instead of 212.

cipher places.) which would give the nearest approach to the present Fahrenheit degree.

The Committee has thus endeavored to unfold with as much conciseness as was compatible with a clear presentation of the subject, what it regards as the best possible method of fulfilling all the varied and diffcult conditions required in an acceptable system of weights and measures, as well as the most effectual means of promoting that great desideratum of international commerce, an ultimate uniformity of standards among the nations of the earth. The serious and radical defects of our existing systems have been briefly noticed, and from the experience thus acquired the essential and practical wants of the community have been incidentally pointed out. As the result of this investigation, we can propose no solution of the problem excepting that we have here displayed; nor are we able to discover the counterbalance of any disadvantages which would follow the adoption of this plan, save that of the disturbance and confusion necessarily consequent upon every change, and which must form the price of every valuable reform.

If it be urged that the introduction of still another system of weights and measures, and one having no common unit with either the French, or the English system, would be only adding to the existing diversity of standards, instead of tending to that great scheme of uniformity so cherished by the philanthropist, we have to reply, that, if the system proposed be really of all others the best adapted to the needs, not only of one, but of all nations, then is the prospect of a general uniformity most reasonably to be anticipated *from* its introduction. If neither the metrology of England, (which is also ours,) nor yet that of France, is ever likely to obtain a universal conquest, some better scheme alone remains to give us a hope of ultimate success. Such a scheme we think is here presented. Founded upon the simplest and yet most comprehensive basis, it contains nothing that could be regarded as in any respect peculiar to one locality or latitude, or more suitable for one nation than for any other. Encumbered by no abstruse nomenclature, it aims at no superfluous verbal uniformity, but leaves each people to employ such designations of its units, as may appear to each most easy and familiar.

Mr. Adams, after his unequalled analysis of the English system of measures, in view of its close agreement with our own, discountenances all attempts at a premature innovation. Without approving in his report of the introduction of the French system, he thinks it would afford the best prospect of securing "uniformity;" and remarks, "were it even possible to construct another system on different principles, but embracing in equal degree all the great elements of uniformity, it would still be a system of diversity with regard to France, and all the followers of her system. And as she could not be expected to abandon that which she has established at so much expense, and with so much difficulty, for another possessing, if equal, no greater advantages, there would still be two rival systems with more desperate chances for the triumph of uniformity."

We must avow our belief, on the contrary, that provided a new system could be framed, which *had* demonstrably "greater advantages" than her own, France would be among the first of nations to hail its advent and to welcome its adoption. A nation to which belongs the honor and the glory of having been the first to conceive a great project of international metrology, the first to invite the fraternal co-operation of other powers, and the first to work out with unwearied science, skill and labor, a comprehensive organization of that ideal metrology — unrivalled in its philosophy and symmetry — cannot be the last to appreciate any real improvement of that economy; or to submit to any sacrifice which should promote the realization of such improvement. Nor could the entire abandonment of that which has cost so much, be accounted too great a sacrifice, if only through it could be accomplished that magnanimous design to which it owed its origin. It would have to be looked upon as a costly but invaluable experiment — as a great and necessary progression to an end, by which alone was rendered possible any higher attainment. The system here elaborated is but a development of *that.*

A project which contemplates the entire subversion of the existing arithmetic, with its immense stores of fact and formula, is certainly a most startling proposal; and is one which will doubtless

be regarded by the majority of persons as a scheme chimerical and impossible. We are impressed with a calm conviction that it does not even offer any real difficulty. The enormous labor of re-construction involved, we seek not to deny or to underrate. But this is a trouble which must always be commensurate with the greatness of the reformation. This necessity would, however, most probably stimulate to the development and perfection of that most useful ally, the calculating machine. Rendered simpler in its construction by the very system which should require its services, and made popular and general by the new demand, it seems not improbable that a single century of the octonary empire would place the world on a higher platform than it would even reach without it. Such has been the usual history of difficulty and of success. We believe that a national government has but to *will* it, to ensure its establishment; and after the first impediments of custom were surmounted, we nothing doubt, that the facility and manifold conveniences of the new *regime* would form its most powerful support, and its surest recommendation to popular favor.

If the octonary system have the germ of vitality, which we have here imagined, its adoption by any one of the great nations of Christendom would as surely pave the way to its universal prevalence, as did the introduction of the Hindoo notation, and of the Gregorian calendar. Nor are the obstacles which so long delayed those great reforms, either as numerous or as serious at the present day, as they were in by-gone centuries. The tone and temper of the times,— intellectual, moral and political — differ widely from those of our ancestors; and in our common school system we have a moral mechanism for the inoculation of new truth, untried and unknown in all past ages.* Whenever the octonary numeration

* In the interesting Report made to the Secretary of the Treasury, Dec. 30, 1856, by Prof. Bache, Superintendent of Weights and Measures, it is well remarked, in relation to the facility of introducing a decimal system, that " One generation would nearly suffice to effect this change, if, as in Holland, the new weights and measures were introduced through the schools. The children of the country becoming familiar with them in the primary schools, seeing the actual material standards of length, capacity, and weight at frequent and stated times in early youth, and retaining that familiarity as they passed into the higher schools, would be readily prepared for their universal use when reaching mature life."

should be definitely established by political authority, we would immediately have all young children instructed for a year or two, only in the octonary arithmetic, — as furnishing the easiest and most rational introduction to the knowledge of figures. And not until after a complete mastery of this arithmetic, should they be taught the use of decimals, — still required for a considerable period to enable reductions to be made from the old style to the new. This would be attended with no more labor than is the additional study now of ordinary Algebra; while in the distinctive languages of the two scales would be found a safeguard against all danger or difficulty, in confounding the one value with the other.

The economy of time and labor which the system of octonary computation would infuse throughout the myriad commercial details daily entering into the life of a busy and enterprising people, cannot be estimated, and could not easily be exaggerated. The popular wonder would be no smaller under the daily workings of this wiser system, that decimals could have prevailed so many centuries, — than is our wonder now that the demands of trade could possibly have been satisfied by the awkward and complex Roman scale of numeration.

The objections naturally brought against any disturbance of the existing order of accountancy, (backed on the other hand by the indolent and dilatory plea that we and our ancestors from earliest time have found it to answer quite "well enough,") are precisely those which have uniformly opposed and retarded the introduction of every improvement. We are informed by Sir John Bowring, in his interesting sketch of the Exchequer system of England, that in quite recent times, Lord Granville strongly resisted the abolition of the Latin phraseology, and the substitution of the Hindoo numerals for the Roman, in the keeping of the public accounts, on the ground that the continuance of the accustomed system was necessary to preserve the comprehension of preceding records! *

* "It is indeed scarcely credible, that the perplexing and entangled manner of keeping accounts by the Roman numerals in the same barbarous style which was practised before the Norman Conquest, was maintained at the Exchequer almost down to the present day; and the introduction of the English language and the Arabic num-

The only question upon the subject we can acknowledge as worthy of discussion, is that which regards the beneficial character of the revolution. "Is, or is not, the change proposed, a real improvement?" If it be, — if it be not only an improvement, but of all projected schemes the truest, wisest, best, — then we assert the bolder logic — *its adoption is only a question of time!* Prejudice, timidity, or indolence, — insensibility to the interests of the future, or superstitious reverence for the grey-haired follies of the past, may each or all oppose their ineffectual resistance; they may indeed postpone for a century or two, the benefit to be enjoyed, — they may indeed throw in the scale the added labor of accumulated work to be undone, — but what is "truest, wisest, best," shall surely, in the end, secure its empire.

To the objection urged by some, that the advantages to result are too remote, and that even were the new arithmetic now inaugurated, the present generation could not expect to have the full and peaceful enjoyment of its alleged conveniences, we would reply that such has been the case with every really great reform. We are in possession now of many priceless blessings, whose first and feeble preparations were planned in former, un-enjoying ages. Shall we reap the rich fruits grown from the unselfish providence of ancestral culture, and shall posterity be less favored? Patriotism and humanity reject the doubt. We believe the octonary algorithm to be pregnant with such great and wide-spread benefits, — benefits to extend throughout all coming time, that we should estimate its acquisition cheaply purchased by whole generations of transitional confusion.

The measure we have thus imperfectly advocated in this Report, is by no means a new one. It is an incident of the highest interest and moment in the reign of that distinguished monarch,

erals was successfully resisted by no less a personage than Lord Granville, on the ground that if the barbarous usages of our ancestors were reformed, it would be difficult to understand the accounts, and the records of departed time; and hence he argued for the necessity of perpetuating a system of complication, confusion, and imperfection, not on the common plea of the superior wisdom of our ancestors, but in full acknowledgment and appreciation of the ignorance of the custom which was originally instituted, and which had continued to reign triumphant among the Exchequer records." (*Bowring's Decimal System, chap.* 7, *p.* 124.)

Charles XII. of Sweden, that he not only contemplated the introduction of an octonary arithmetic, but that he commissioned Swedenborg (at that time celebrated for his scientific and mathematical attainments,) to draw up the necessary details of the plan for establishing this system, — together with an octonary scale of weights, measures, and coins throughout his kingdom.* It appears that the premature death of the king very shortly afterward alone prevented the consummation of this most sagacious and philosophic enterprise. But for this untoward circumstance, this admirable mechanism would have thus been put into practical operation nearly a century and a half ago! Had it proved as successful as there is every reason to suppose it would, — who can estimate the influence this ingrafting would have had upon the present mathematical condition of Europe? Might we not now have been in the full and assured enjoyment of that happier system? The subject of this improved numerical notation had doubtless often occupied the minds of mathematicians, long before this time; but we believe that this is the first occasion on which a deliberate and well designed attempt was ever made to give it a practical existence and establishment. As such, it is an event of no trivial importance, and must be regarded as ever memorable in the history of arithmetical reform.

In contemplating the practical working of this untried system, and forming an estimate of the character of the change required in the popular habits of thought, comparison, and judgment, there can be no doubt that the octonary scale could be generally introduced with far greater facility, and made thoroughly familiar in a much shorter time, in its application to the divisions of money, weight, and measure, than it could be in its more abstract application to the operations of universal numeration. We believe, therefore, that in advance of the arithmetical reformation, it would be found highly expedient to introduce the simple and convenient system of weights and measures here proposed, as the best preparation for the successful introduction of the other.

Even were the octonary arithmetic, (with all its own intrinsic

* See note D — appended to this Report.

excellences) not to be adopted, we still urge that these measures would be worthy of an independent establishment. After the variety of arithmetical reductions to which we are now accustomed under our present incongruous tables, the uniform reduction of a single scale, which would alone be required in the new order, would give a very great simplification and relief; and would in every probability be found upon the whole to entail less inconvenience than that which would remain, with even the perfect decimalization of our various measures. So that even under the disadvantages of a decimal dispensation, we believe it could easily be shown that our new system would still, in view of all the circumstances, be the "best possible" one for popular use, and would most completely furnish the elements of a perfect uniformity.

We have presented the results of our most mature deliberation. With these suggestive representations, our province terminates. As indicated in an early part of this Report, only the authority of Government can give validity to such proposals of reform, whatever their theoretical excellence. For the still more extended and desirable improvement of an *international* uniformity, (the aspiration and the labor of so many among the world's best intellects,) a concert of action among the leading maritime powers, affords the only means of realization. Such a conference among nations having so many fraternal ties, seems called for by every principle of wise philosophy, of generous humanity, and of an enlightened sense of common interest; and surely cannot be regarded, at this day, as a visionary or illusive expectation.

Forty years have now elapsed since the resolution of Congress calling upon the Secretary of State for information respecting the standards of the several States, "and relative to proceedings in foreign countries, for establishing uniformity in weights and measures; together with such a plan for fixing the standard of weights for the United States, as he might deem most proper for their adoption," drew forth the masterly Report of Mr. Adams, from which, in recognition of its standard and authoritative character, we have so liberally quoted.*

* The Resolution of the House of Representatives was passed December 14th, 1819. The Report (extending to 145 pages,) was submitted February 22d, 1821. So that this

With a comprehensive view of all the bearings of the subject, Mr. Adams looked eagerly forward to the estȧblishment of a uniform system of metrology among all enlightened nations. "Would it not be strange," he asks, "if, while mankind find it so easy to attain uniformity in the use of every engine adapted to their mutual destruction, they should find it impracticable to agree upon the few and simple, but indispensable instruments of all their intercourse of peace and friendship and beneficence; that they should use the same artillery and musketry, and bayonets and swords, and lances, for the wholesale trade of human slaughter; and that they should refuse to weigh by the same pound, to measure by the same rule, to drink from the same cup, to use, in fine, the same materials for ministering to the wants and contributing to the enjoyments of one another?"

And yet, with a rare prudence and moderation, believing the time was not then come for any decisive movement, he avoided the tempting occasion offered by the language of the resolution of inquiry, to propose some model scheme; and urged upon Congress, with equal earnestness and wisdom, the avoidance of all attempts at partial and precocious reformation. "In contemplating so great, but so beneficial a change," says he, "as the ultimate object of the proposal now submitted to the consideration of Congress, it is supposed to be most congenial to the end, to attempt no present change whatever in our existing weights and measures. . . . . . If there be one conclusion more clear than another, deducible from all the history of mankind, it is the danger of hasty and inconsiderate legislation upon weights and measures. . . . . All trifling and partial attempts of change in our existing system, it is hoped, will be steadily discountenanced and rejected by Congress; not only as unworthy of the high and solemn importance of the subject, but as impracticable to the purpose of uniformity, and as inevitably leading to the reverse, to increased diversity, to inextricable confusion." And accordingly as the result of his laborious investigations, he submits this simple conclusion: "The plan which

elaborate and voluminous work, involving a vast amount of research and of reflection, was prepared in the short space of one year and amid the pressing duties and labors of his high official position. It forms No. 109 of the House Documents of the Second Session of the 16th Congress.

is thus, in obedience to the injunction of both Houses of Congress, submitted to their consideration, consists of two parts, the principles of which may be stated: 1st. To fix the standard, with the partial uniformity of which it is susceptible for the present, excluding all innovation. 2d. To consult with foreign nations, for the future and ultimate establishment of universal and permanent uniformity." In a preceding part of his Report, he remarks: "A single and universal system can be finally established only by a general convention, to which the principal nations of the world shall be parties, and to which they shall all give their assent. To effect this, would seem to be no difficult achievement. It has one advantage over every plan of *moral* or *political* improvement, — there neither is nor can be any great counteracting *interest* to overcome. The conquest to be obtained is merely over prejudices, usages, and perhaps national jealousies. The whole evil to be subdued is diversity of opinion with regard to the means of attaining the same end. To the formation of the French system, the learning and the genius of other nations did co-operate with those of her native sons. The co-operation of Great Britain was invited; and there is no doubt that of the United States would have been accepted, had it been offered. . . . . But this agreement can be obtained only by consultation and concert. It is therefore respectfully proposed as the foundation of proceedings necessary for securing ultimately to the United States a system of weights and measures which shall be common to all civilized nations, that the President of the United States be requested to communicate through the ministers of the United States in France, Spain, and Great Britain, with the governments of those nations upon the subject of weights and measures, with reference to the principle of uniformity as applicable to them."

Thirty-six years later than the date of these suggestions, Mr. Bache, in a report upon the same subject, which may justly be regarded as the natural sequel to the earlier one of Mr. Adams, thus renews this great and important project.

"The first part of Mr. Adams's plan has, as far as legal standards are concerned, in a great degree been accomplished; but the second part, that which recommends the consultation with foreign

nations for the ultimate establishment of universal uniformity, remains yet to be acted on. The importance of the subject cannot but be deeply felt by every one truly interested in the universal brotherhood and substantial improvement of the human race. France was the first to take an important step in the matter, in the projection and final accomplishment of the great work of the measurement of an arc of the meridian passing through her territory, thus giving an ultimate natural standard to which all measures of extension might be referred. To this work she gave the important and universal character which it deserved by inviting the co-operation of all nations. The subsequent adoption of the metrical system, with the metre, the forty millionth part of the circumference of the earth, as the unit of linear measurement, was a great era in the progress of uniformity. True, the progress of science has shown that the metre is not really the forty millionth part of the meridian, as supposed; true that Great Britain has developed an opposition to this national standard which seems almost insuperable, and has been followed by the continental nations in this opposition. The first fact does not necessarily imply a change of unit, though it does point to the failure to realize its true extent; but the second seems conclusive in regard to the necessity for new efforts at international uniformity. . . . . . And has not the time arrived, in the general progress of commercial and international intercourse, and the rapid advance of our own country in science, wealth, and power, when her voice should be heard in an important matter like this? Should not Congress make the proposition to all nations to meet by their representatives, and consult for the purpose of establishing permanent and universal uniformity of weights and measures? Such action could not fail to meet with a response due to the greatness of the subject; and, if the great object be attained, to lead to results productive of vast and lasting benefit to the human race."

These propositions, the dictates of an enlightened sentiment of patriotism as well as of philanthropy, commend themselves to the wisdom of those who under our National Constitution, alone have power "to fix the standard of weights and measures."

Respectfully submitted,

ALFRED B. TAYLOR.

NOTE A.

"The triumph of the art of calculation, and that to which mainly the modern system of numeral computation, owes its perfection, consists in the 'device of place,' by which all necessity for distinguishing the nature of the units signified by any symbol is superseded. Like many other inventions of the highest utility, this, when known, appears to arise so naturally and necessarily out of the exigencies of the case, that it must excite unqualified astonishment how it could have remained so long undiscovered. . . . . . That the honor of the invention of a system which produced such important effects as well on the investigations of science, as in the common concerns of commerce, should be claimed by many contending nations, is what would naturally be expected. . . . . . All Arabian authors on Arithmetic appear to agree that the first writer of that country upon this system of arithmetic, was Mohammed ben Muza, the Khuwarezmite, who flourished about the year 900. This writer is celebrated for having introduced among his countrymen many important parts of the science of the Hindoos, to the cultivation of which he was devotedly attached; and among other branches of knowledge thence derived, there is satisfactory evidence that this species of arithmetic was one. From the time of Mohammed ben Muza, the figures and modes of calculation introduced by him were generally adopted by scientific writers of Arabia, although a much longer period elapsed before they got into common popular use, even in that country. They were always distinguished by the name *Hindasi;* meaning the Indian mode of computation. . . . . At the beginning of the eleventh century, the use of the Arabic notation had become universal in all the scientific works of Arabian writers, and more especially in their astronomical tables. The knowledge of it was of course communicated to all those people with whom the Moors held that intercourse which would lead to a community of scientific research. In the beginning of the eleventh century, the Moors were in possession of the southern part of Spain, where the sciences were then actively cultivated. In this way the use of the new arithmetic was received into Europe first in scientific treatises. A translation of Ptolemy was published in Spain in 1136, in which this notation was used; and after this period it continued in general use for the purposes of science. Notwithstanding the knowledge and practice of this superior notation by scientific men, the Roman numerals continued to be used for purposes of business and commerce for nearly three centuries; and it was only by slow and gradual steps that the improved notation prevailed over its clumsy and incommodious predecessor. The first attempt to introduce it for the purposes of commerce, was made by a Tuscan merchant, Leonardo Pisano, in 1202. Having travelled in Barbary, he there learned the method of Hindoo arithmetic; and struck with its superiority over that to which he had been accustomed, he determined that his countrymen should no longer be deprived of the benefits of it. He accordingly published his treatise in the Latin language; in which he professes to deliver a complete doctrine of the numbers of the Indians. . . . . A considerable period, however, was necessary to introduce this system into the common business of life. The extensive commerce maintained by the Italian States directed their attention to the subject

at an earlier period than other nations; and although for scientific purposes, the date of the introduction of the Arabic numeration into Spain is earlier than that of its appearance in Italy, yet its use for the common business of life, prevailed at a much earlier period among the Italian States, than in any other nation of Europe." (*Lardner's Treatise on Arithmetic*, book 1, ch. 2.)

The Hindoo numerals are found in various manuscripts of Italy, bearing the dates 1212, 1220, 1228. But none are found in England till nearly two centuries later. Chaucer, the poet, who died in 1400, alludes to them in one of his poems, as "the *figures newe*."

According to Sir John Bowring, ("Decimal System:" pages 23–30,) the first calendar in the English language in which the Hindoo numerals are employed, bears the date of "1431," and the earliest date known on a tombstone in these figures is "1454;"—the tombstone being that of "Elen Cook," in the church at Ware. The first English book which bears its date in these figures, is the "*Rhetorica Nova*, Gulielmi de Saona, 1478." And in seals, only one example has been found anterior to the sixteenth century, which bears the date 1484. "The Roman figures lingered longer in England," adds Bowring, "than in any other part of the European world; having found an asylum in the dark and dull regions of the Exchequer," (page 26.) "It is indeed scarcely credible, that the perplexing and entangled manner of keeping accounts by the Roman numerals, in the same barbarous style which was practised before the Norman Conquest, was maintained at the Exchequer, almost down to the present day. . . . . . In addition to this strange and absurd system of Exchequer book-keeping, tallies continued to be used down to the year 1782. It was only in the year 1831, that the Committee on Public Accounts, of which I was the Secretary, recommended the utter and complete abolition of the ancient system, and the adoption of the Indian numerals. It was in consequence of this change, that in the year 1835, the *tallies* were ordered to be burnt; a conflagration which led to the destruction of both Houses of Parliament,—the Exchequer in which the tallies were kept, having formed a part of the ancient edifice of St. Stephens." (*Sir John Bowring's Decimal System*, pages 124–125.)

Delambre regards it as a fact humiliating to the pride of human genius, that the discovery of the true notation of number by nine digits and zero should have escaped the sagacity of the illustrious geometers and mathematicians of ancient Greece. "The Hindoos," says Peacock, "consider this method of numeration as of divine origin; 'The invention of nine figures with the device of place being ascribed to the beneficent Creator of the universe.' Of its great antiquity amongst them," he continues, "there can be no doubt, it having been used at a period certainly anterior to all existing records." (*Encyclopedia Metropolitana.*) It can be traced back with certainty at least four centuries before its appearance among the Arabs; and as Lardner well remarks, since "none of these Hindoo authors claim either for themselves or their predecessors, the invention of this method of numeration, but always mention it as being received from the Deity, we may infer that it was practised in that country beyond the limits even of tradition." The Indian origin of our numerals being thus so well established, there is a manifest

impropriety in continuing to designate them as the "Arabic figures," as is constantly done in our school Arithmetics. Let us give honor where honor is due.

NOTE B.

It is remarkable that this binary system, according to the opinion of many, was used in China, four thousand years ago, by Fohi, the founder of the empire. A tablet of great but unknown antiquity, called the Cova of Fohi, marked with a series of variously broken lines, and held in superstitious reverence by the Chinese, as containing the mystery of a divine wisdom, has been found to be completely deciphered by the notation of binary arithmetic. When Leibnitz had extensively circulated his scheme or invention through the various scientific journals, and by means of his own correspondence,— it appears to have found its way even to China, and to have attracted the attention of a Jesuit Missionary at Pekin, named Bouvet. This ecclesiastic, engaged at the time in the study of the Chinese antiquities, discovered, and immediately communicated to Leibnitz with much exultation and enthusiasm, the surprising fact that his system furnished a perfect key to the mysterious lines upon the ancient Cova — hitherto inscrutable, or interpreted only by the speculations of the most extravagant mysticism. The lines of Fohi are arranged in an octagonal form, so as to make the ends approach; — each set of the eight series being disposed on a side of the octagon. These lines transferred from the Cova tablet, and placed in a straight line, are here represented. The row of figures in front expresses the value of each compound symbol, the other figures, which represent the binary notation, manifestly exhibiting a perfect correspondence with the symbols throughout.

0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

" These figures of eight cova," says Mr. Peacock, (in the *Encyclopedia Metropolitana*) " are held in great veneration, being suspended in all their temples, and though not understood, are supposed to conceal great mysteries, and the true principles of all philosophy, both human and divine."

This inscription is exceedingly interesting as exhibiting a true example of that philosophic notation, the device of the cipher — and the determination of value by place. The absence of any other traces of such a notation in China, and its well-known antiquity in India, where it had been so fully elaborated, would lead to the suspicion that it was to this latter country that Fohi was indebted for this curious record of ingenious thought. It appears that Bouvet was fortunate enough to find subsequently, a Great Cova, in which these markings were carried to a period eight times the extent of the Small Cova. In the *Edinburg Encyclopedia* — (article " Arithmetic ") it is stated in reference to this subject, that " Father Bouvet, who first suggested this explanation, and communicated it to Liebnitz, afterwards procured, during his residence in China, the *Great Figure of Fohi*, which extends as far as 64. The exact coincidence which he still found to prevail between the combinations of these lines and the figures of the binary notation, left no doubt with regard to the justness of his conjecture; and we

cannot help remarking that the restitution of the true sense of those characters, after so long an interval of time, is a very singular fact in the history of science."

NOTE C.

It is interesting to trace the history of the gradual development, in modern times, of the grand but difficult project of obtaining from nature a constant and universal standard of length. It is obvious that no such objects of ultimate reference as the human foot, or arm, or cubit, or as "thirty-six barley corns round and dry," can be regarded as natural standards, since they are wholly useless for the purpose of any precise determination. And all measures derived from them are purely arbitrary, as their authority is obtained from positive enactment, merely, and not from any agreement with their nominal originals. Hence it is not at all surprising that "cubits" and "feet" come to signify anything the civil power may enact; the former of these denominations ranging through every gradation of value, from the *covid* of 14½ inches to the royal Egyptian *cubit* of 25½ inches, and the latter from the Pythic foot of 9¾ inches, to the Geneva foot of 19 inches. Nor would it ever be possible from such sources, to reproduce a lost standard, with even the rudest approach to exactness. The necessity indeed for absolute and invariable units of measure, is only felt in a highly advanced condition of science, and of civilization. As Mr. Adams has well remarked, "For all the uses of weights and measures in their ordinary application to agriculture, traffic, and the mechanic arts, it is perfectly immaterial what the natural standard to which they are referable may be. The foot of Hercules, the arm of Henry the First, or the barley-corn is as sufficient for the purpose as the pendulum, or the quadrant of the meridian." (*Report to Congress.*)

"The first attempt at fixing such a standard as should be accurate and universal, both as to place and time, is due to the inventive genius of the celebrated Huyghens. That philosopher demonstrated that the times of the vibrations of pendulums depend on their length only. . . . Hence he conceived that the pendulum might afford a standard or unit for measures of length." (*Edinburgh Review*, vol. 9, p. 373.) It was in his "Horologium Oscillatorium" (published about 1670,) that Huyghens proposed the use of the seconds pendulum as a universal and perpetual measure; this length to be divided into three equal parts; and this third part (about 13 inches) to be called the *horary foot.*

The celebrated Picard, who first measured from Paris to Amiens in 1669, an arc of the meridian in France, making the degree equal to 68.945 miles, (a measurement memorable as having furnished Newton with the means of verifying his grand theory, incapable of determination from the pre-existing data,) also proposed in 1671, in agreement with the idea of Huyghens, that the pendulum beating seconds should be adopted as the unit of length. Picard has the merit of having first thrown out the suggestion, that the diurnal rotation of the earth ought to affect the oscillations of the pendulum, and that it ought to vibrate more rapidly toward the poles than toward the equator. He accordingly tried the pendulum at Uranibourg, at Paris and at Cette, but

was not fortunate enough to discover any sensible difference. Roemer also found the length the same at London.

Richer, however, in the same year, 1671, or early in 1672, while engaged in the duties of his commission at Cayenne, on observing the length of the seconds' pendulum at this place (lat. 4°, 56′ north of the equator,) found it sensibly shorter than at Paris, (48°, 50′ north,) the difference being about a line and a quarter. Richer's discovery that the pendulum varied in length with the latitude, deprived it of that uniform character considered so necessary in a linear *standard.*

The Abbe Gabriel Mouton, a distinguished mathematician who flourished at the same time, appears to be the first who suggested a measure derived from the earth. He proposed, almost simultaneously with the publication of Huyghens, a *decimal system of measures*, based on the value of a minute of arc, as derived from Riccioli's length of a degree. This minute of the degree he called a *miliare*, the thousandth part of which he called a *virga;* equal to 5 feet, 4⅛ inches. We have here the germ of the present French Metrology.

Cassini, who in 1718 repeated the measurements of a meridian made by Picard, (extending his arc however further south, namely, from Paris to Dunkirk, and making the degree 69.119 miles) proposed the earth's radius as the unit of length. He afterwards in his book *De la Grandeur de la Terre*, proposed as the unit, the six thousandth part of a minute of a degree of a great circle of the earth; a measure very nearly equal to the foot.

In 1748, M. de la Condamine, (who had recently returned from measuring a degree at the equator in Peru,) in a memoir read before the Academy of Sciences, resumed the idea of the pendulum as the unit of length; proposing that it should be taken as beating seconds at the equator, as the most notable line of latitude, and as one likely to avoid all the prejudices which might arise from national jealousy, were the latitude of any particular place selected. We see from this, the anxiety felt to secure a standard which might be common and uniform among nations.

In 1790, Talleyrand proposed to the constituent Assembly of France, that in view of the great diversity and confusion in the weights and measures of the country, a commission should be appointed for the purpose of consulting with a similar commission from the English Government, upon the subject of establishing a uniform international system of metrology, founded upon a single and universal standard. The proposal alluded to the only two natural standards which presented themselves, viz., the measure of the earth and the pendulum, and expressed a decided preference for the latter. The result of this movement was the appointment of Borda, Lagrange, Laplace, Monge, and Condorcet, as commissioners to examine into and report upon the subject. After a careful consideration of the three plans submitted, namely, the pendulum, a quarter of the equator and a quarter of the terrestrial meridian, they very judiciously agreed in decidedly recommending the latter; regarding the pendulum as an unsuitable standard, whether taken at forty-five degrees of latitude or at the equator.

The attempt to enlist the co-operation of England proved abortive. "The operation of changes of opinion there," says Mr. Adams, "is slow — the

aversion to all innovations deep. More than two hundred years had elapsed from the Gregorian reformation of the calendar, before it was adopted in England. . . . After a succession of more than sixty years of inquiries and experiments, the British parliament have not yet acted in the form of law." (*Report to Congress.*)

It was just five hundred years after the statute of 17th Edward II., (A. D., 1324,) enacted that "three barley corns round and dry, make an inch — twelve inches make a foot," &c., before any change was made in the legal definition of the foot. By act of 5th George IV, c. 74, (1824,) it is declared "the standard yard is the distance between the centres of the two points on the gold studs in the straight brass rod now in the custody of the Clerk of the House of Commons, whereon is engraved 'standard Yard 1760,' the brass being at the temperature of 62 degrees by Fahrenheit's thermometer." [This brass standard yard was executed by Mr. Bird, a celebrated optician in 1760.] "The Yard, if lost, defaced, or otherwise injured, may be restored by comparing it with the pendulum vibrating seconds of mean time in the latitude of London, in a vacuum, on the level of the sea, the yard being in the proportion of 36 inches, to 39.1393 inches of the pendulum." This was the first attempt to refer the English foot to a natural standard.

Ten years afterwards, or in 1834, the contingency provided for by this statute actually occurred by the burning of the Houses of Parliament; in which conflagration the celebrated brass standard of Bird was destroyed. Although the only actual legal standard was thus lost, no attempt was made to restore it by the pendulum, as provided by law; but the mean of twelve different standards, including one belonging to the Royal Astronomical Society, which had been constructed and carefully compared with Bird's Imperial Standard, a short time before its destruction, was selected as giving the nearest approximation to the legal standard yard.

A commission was appointed by the British government, in 1838, "to consider the steps to be taken for the restoration of the Standards of Weight and Measure." The Commissioners in their Report, made in 1841, say: "We are of opinion that the definition contained in the Act 5, Geo. IV, c. 74 ss. 1 & 4, by which the standard yard and pound are declared to be respectively, a certain brass rod and a certain brass weight therein specified, is the best which it is possible to adopt. Since the passing of the said act, it has been ascertained that several elements of reduction of the pendulum experiments therein referred to are doubtful or erroneous; thus the reduction for the weight of air was erroneous; the specific gravity of the pendulum was erroneously stated, the faults of the agate plates introduced some degree of doubt, and sensible errors were introduced in the operation of comparing the length of the pendulum with Shuckburgh's scale, used as the representative of the legal standard. It is evident therefore that the course prescribed by the act would not necessarily reproduce the length of the original yard. Several measures however exist, which were most accurately compared with the former standard yard. And we are fully persuaded, that with reasonable precautions, it will always be possible to provide for the accurate restoration of standards by means of material copies which have been carefully compared with them,

more securely than by reference to any experiments referring to natural constants." And the report concludes by recommending "that the standard of length be defined by the whole length of a certain piece of metal or other durable substance, supported in a certain manner, at a certain temperature; or by the distance between two points or lines engraved upon the surface of a certain piece of metal or other durable substance, supported in a certain manner and at a certain temperature; but that the standard be in no way defined by reference to any natural basis, such as the length of a degree of meridian on the earth's surface in an assigned latitude, or the length of the pendulum vibrating seconds in a specified place . . . . That the standard of weight be defined by a certain piece of metal or other durable substance," &c.

It thus appears as the result of this last Commission in England, that the people of that country are disposed to abandon all attempts at obtaining a natural standard, and to recur to the authority of an arbitrary rod or piece of metal, whose length has been derived from prescriptive custom. It should be considered, however, that after a natural standard has been obtained, we still have all the means of its material perpetuation, suggested in the Commissioners' Report. And no foreign community is ever likely to accept as an authoritative unit of measure, a certain brass rod manufactured in England, and incapable of any more precise definition.

In addition to the difficulties of obtaining from the pendulum the re-construction of a lost standard, as above indicated, it is not unimportant to note that there is an original uncertainty in the determination of its length, of nearly the thousandth part of an inch. "We cannot venture to say that the clock's rate in a given day, can be determined certainly to within one-tenth part of a second, although the comparisons have been made at an interval of twenty-four hours. Seeing then that the *free* pendulum is compared with the clock only over a small fraction of the day, it is a great deal to expect that *its* daily rate can be ascertained to within one second of time. A change of one second per day in the rate of a clock, corresponds to a change of $\frac{1}{43200}$ in the length of the pendulum, which is about $\frac{1}{1100}$ of an inch, or $\frac{1}{43}$ of a millimetre; and therefore we may regard this distance as indicating the probable limit of exactitude." (*Encyclopedia Britannica*, 8th edition, vol. 17, p. 384, article "Pendulum," by Edward Sang.)

NOTE D.

The only account we have been able to obtain of the important movement of Charles XII toward superseding the decimal by the octonary system, throughout Sweden, is that contained in a volume entitled "A Compendium of the Theological and Spiritual Writings of Emanuel Swedenborg," (royal octavo,) published at Boston by Crosby & Nichols, 1854. In the life of Swedenborg, prefixed to the "Compendium," it is said, "In 1719 he published four works; first, '*A Proposal for fixing the value of Coins and determining the Measures of Sweden, so as to suppress fractions, and facilitate Calculations.*' After which he was commanded by his Sovereign to draw up an Octonary Computus, (a mode of computing by eighths,) which he completed in a few days, with its application to the received divisions of Coins, Weights and Measures; a disquisition on Cubes and Squares, and a new and easy way of

extracting Roots; all illustrated by appropriate examples." (*Life*, p. 9.) As Swedenborg devised for his "Octonary Computus," both a set of characters, and of new names, we were exceedingly anxious to have enriched this Report with their representation. We have failed, however, to find any clue to these early publications in any of the public Libraries or private collections to which we have had access. The only additional reference to the subject in the volume above referred to, is contained in a letter from Swedenborg to M. Nordberg, written after the death of Charles XII, which appears to detail the monarch's first conception of the project of a reformation in the popular system of numeration. An extract, giving all that relates to the subject of octonary computation, is here copied.

Letter of M. Swedenborg, Assessor of the Board of Mines, to M. Nordberg, Author of the History of Charles XII.

"SIR: — As you are now actually engaged upon the Life of Charles XII, I avail myself of the opportunity to give you some information concerning that monarch, which is perhaps new to you, and worthy of being transmitted to posterity. . . . . . Conversing one day with the king upon arithmetic, and the mode of counting, we observed that almost all nations, upon reaching ten, began again; that those figures which occupy the first place, never change their value, while those in the second place were multiplied ten-fold, and so on with the others; to which we added that men had apparently begun by counting their fingers, and that this method was still practised by the people; that arithmetic having been formed into a science, figures had been invented, which were of the utmost service; and, nevertheless, that the ancient mode of counting had been always retained, in beginning again after arriving at ten, and which is observed by putting each figure in its proper place.

The king was of opinion that had such not been the origin of our mode of counting, a much better and more geometrical method might have been invented, and one which would have been of great utility in calculations, by making choice of some other periodical number than 10. That the number 10 had this great and necessary inconvenience, — that when divided by 2, it could not be reduced to the number 1, without entering into fractions. Besides, as it comprehends neither the square, nor the cube, nor the fourth power of any number, many difficulties arise in numerical calculations. Whereas, had the periodical number been 8, or 16, a great facility would have resulted, the first being a cube number of which the root is 2, and the second a square number of which the root is 4; and that these numbers being divided by 2, their primitive, the number 1 would be obtained, which would be highly useful with regard to money and measures, by avoiding a quantity of fractions. The king, after speaking at great length on this subject, expressed a desire that we should make a trial with some other number than 10. Having represented to him that this could not be done unless we invented new figures, to which also names altogether different from the ancient ones must be given, as otherwise great confusion would arise, he desired us to prepare an example in point. We chose the number 8, of which the cube root is 2, and which being divided by 2, is reduced to the primitive number 1. We also invented new figures, to which we gave new names, and proceeded according to the ordinary method; after which we applied them to the cubic calculations, as well as to money, and to measures. The essay having been presented to the king, he was pleased with it." (*Appendix to Life, &c., pp.* 123, 124.)